Target recognition of log-polar ladar range images using moment invariants

Wenze Xia a,b, Shaokun Han a,*, Jie Cao a,b, Haoyong Yu b

a School of Optoelectronics, Beijing Institute of Technology, Beijing, 100081 China
b Department of Biomedical Engineering, National University of Singapore, Singapore, 117575 Singapore

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The ladar range image has received considerable attentions in the automatic target recognition field. However, previous research does not cover target recognition using log-polar ladar range images. Therefore, we construct a target recognition system based on log-polar ladar range images in this paper. In this system combined moment invariants and backpropagation neural network are selected as shape descriptor and shape classifier, respectively. In order to fully analyze the effect of log-polar sampling pattern on recognition result, several comparative experiments based on simulated and real range images are carried out. Eventually, several important conclusions are drawn: (i) if combined moments are computed directly by log-polar range images, translation, rotation and scaling invariant properties of combined moments will be invalid (ii) when object is located in the center of field of view, recognition rate of log-polar range images is less sensitive to the changing of field of view (iii) as object position changes from center to edge of field of view, recognition performance of log-polar range images will decline dramatically (iv) log-polar range images has a better noise robustness than Cartesian range images. Finally, we give a suggestion that it is better to divide field of view into recognition area and searching area in the real application.

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1. Introduction

As theoretical and technical developments continued, ladar has attracted considerable attentions and has been widely applied in every aspect of the modern world [1–3]. In general, ladar can be sorted into two types: scanning type and non-scanning type [4]. Different ladar type has its own merit and demerit [5,6], so it is very difficult to conclude which type will replace another type in the future. Ladar is capable to effectively acquire range images which are closely related to real three dimensional (3D) surface information of target. Hence, target recognition using range image is less sensitive to various inherent problems in two dimensional (2D) images [7], and has been extended to various field [8–10].

The target recognition algorithm based on range images can be classified as local feature representation algorithm and global feature representation algorithm [11,12]. The local feature representation algorithm describes small scale shape features around feature points, and it is not affected by clutter and occlusion. However, this algorithm requires a lot of computation time as well as storage space, and it is sensitive to noise [12,13]. In the field of local feature representation algorithm, many excellent algorithms have been proposed, such as Spin Image(SI) [14], Local Surface Patch(LSP) [15], Intrinsic Shape Signatures(ISS) [16], Variable Dimensional Local Shape Descriptors(VD-LSD) [17], Tri-Spin Image(TriSI) [18], and so on [11]. In [11], there is a comprehensive survey of 3D object recognition based on existing local feature representation algorithms. The global feature representation algorithm does not require a lot of computation time and storage space, and is also less sensitive to noise. However, global feature representation algorithm generally needs complicated image preprocessing, and its characters make it very difficult to solve the occlusion problem [11,19]. In the field of global feature representation algorithm, many excellent algorithms also have been proposed, such as Slice Image [13], Geometric 3D Object [20], Shape Distribution [21], Viewpoint Feature Histogram [22], and so on [12]. In [12], there is a detailed overview about target recognition algorithms based on transform coefficient which belong to global feature representation algorithm. In real application, ladar range image always inevitably contains strong noise [23,24]. Many existing local shape descriptors cannot work effectively with these degenerated range image because of their dependency on dense and smooth full-body sampling points [12]. Therefore,
The development of ladar technique has undergone several decades. However, previous techniques mainly focus on space-invariant detection or approximate space-invariant detection. The research on space-variant detection is comparatively rare. Recently, in [4], our team presented a log-polar 3D imaging ladar system based on micro-opto-electro-mechanical-systems (MOEMS) mirror, and validated its effectiveness by real experiment. Furthermore, in [25], our team proposed a model of log-polar 3D imaging ladar system based on non-scanning principle. To the best of our knowledge, there are no relevant studies covering target recognition of log-polar ladar range images prior to this research. Therefore, in this paper, we construct a target recognition system based on log-polar ladar range images, and fully analyze the effect of log-polar sampling pattern on recognition performance of whole system. In this system, considering the existence of strong noise and requirement of real-time running, Hu moments and affine moments are used as global feature representation descriptor, and backpropagation neural network (BPNN) is used as feature classifier. Eventually, we draw several important conclusions: (i) if combined moments are directly computed by log-polar ladar range images, translation, rotation and scaling invariant properties of combined moments which is necessary for shape descriptor in the object recognition application will be invalid (ii) when object is located in the center of field of view (FOV), recognition rate of log-polar ladar range images is less sensitive to the changing of FOV than that of Cartesian ladar range images (iii) as object position changes from center to edge of FOV, recognition performance of log-polar ladar range images will decline dramatically, and it is not affected for Cartesian ladar range images (iv) recognition rate of log-polar ladar range images is less sensitive to noise than that of Cartesian ladar range images. Moreover, the results also prove that the recognition network trained by log-polar range images has a better recognition performance than that trained by Cartesian range images in the real condition with noise. In the last, we present a suggestion that in the real application FOV should be divided into recognition area and searching area, so that it can balance the conflict between excellent recognition performance and large searching area.

The paper is organized as follows. In Section 2, we firstly review the basic structure of whole target recognition system based on log-polar ladar range images. Then, we show every portion of this system in detail, respectively. In Section 3, several comparative experiments based on ladar range images are performed. In Section 4, several conclusions based on the previous experiments are drawn.

2. Theory

2.1. Target recognition system

Target recognition algorithm based on moment invariants is a very popular algorithm, and it has been successfully applied to target recognition of ladar range image [27,28]. In this paper, the key components of the whole target recognition system based on log-polar ladar range images are summarized in Fig. 1.

In this system, considering the existence of strong noise and requirement of real-time running, combined moments (CM) which contain Hu moments and affine moments are used as global feature representation descriptor, and BPNN is used as feature classifier. The system consists of two processes: BPNN training and recognition. In the BPNN training process, firstly, a series of log-polar ladar range images which are used to construct training sample set are sampled at preset attitude angles. Secondly, all the log-polar ladar range images are converted into CARTESIAN ladar range images. Thirdly, the regions of targets are extracted from all CARTESIAN ladar range images, and their shape features are represented by CM. Finally, all the CM vectors are used to train BPNN classifier. In the recognition process, firstly, a log-polar ladar range image is sampled at random attitude angle. Then its CM vector can be obtained after undergoing the same image preprocessing. Finally, this CM vector is inputted the above BPNN, and the final recognition result depends on the output result of BPNN.

2.2. Coordinate transformation

With the development of imaging technique, human have been trying their best to get higher image resolution. However, higher image resolution is naturally conflicted to larger FOV. In order to address this problem, a log-polar imaging technology, which comes from bionic technology, has been proposed, and it can encode a large FOV with variable spatial resolution [26]. There is a high resolution in the center of FOV and low resolution in the edge of FOV for every log-polar image. Furthermore, log-polar image is rotation invariant and nearly scale invariant [29]. For these reasons, log-polar imaging technology has been widely applied in visual attention, target tracking, egomotion estimation, and 3D perception [30]. Recently, in [4,25], our team had extended log-polar imaging technology to the hardware field of ladar. In this paper, we continue to research the software part of log-polar imaging technology. The mathematical expression of log-polar sampling pattern can be written as [25]:

\[ r_i = \frac{R_0}{1 - \sin\left(\frac{\pi i}{N}\right)}. \]  

(1)

\[ D_i = \frac{2R_0 \sin\left(\frac{\pi i}{N}\right)}{1 - \sin\left(\frac{\pi i}{N}\right)}. \]  

(2)

\[ q = \frac{1 + \sin\left(\frac{\pi i}{N}\right)}{1 - \sin\left(\frac{\pi i}{N}\right)}. \]  

(3)

\[ r_i = q^{1-i} r_i. \]  

(4)

\[ D_i = q^{1-i} D_i. \]  

(5)
\[ R_{\text{max}} = R_0 + \sum_{i=1}^{M} D_i = R_0 q^M. \]

where, \( i = 1, 2, \ldots, M; \) \( R_0 \) and \( R_{\text{max}} \) is the minimum and maximum value of log-polar sampling scope; \( r_i \) is the distance between origin and \( i \)th ring sampling points; \( D_i \) is the diameter of rounded sampling region of \( i \)th ring; \( q \) is the increasing coefficient between adjacent rings; \( M \) is the number of rings of log-polar sampling pattern; \( N \) is the number of sampling points for every ring. From Eqs. (4) and (5), we can find that the distance between two consecutive sampling points and the diameter of every rounded region in the radial direction increase exponentially from center to furthest circumference. The concrete geometry meanings of all the above symbols are shown in Fig. 2.

A typical log-polar sampling pattern is shown in Fig. 3(a), and for no-scanning sampling type every center point of these circles represents the position of every pixel. But for scanning sampling type every center point of these circles represents the position of every scanning point. A log-polar ladar range image is shown in Fig. 3(c). After obtaining log-polar ladar range image, in order to implement target recognition, this image needs to be converted into CARTESIAN ladar range image. The mathematical expression of inverse log-polar mapping can be written as:

Fig. 2. The symbol expression of log-polar sampling pattern.

Fig. 3. Inverse log-polar mapping. (a) log-polar sampling pattern (b) trapezoid-like quadrangle (c) log-polar ladar range image (d) CARTESIAN ladar range image.
where, $x$ and $y$ are the coordinates in the Cartesian space, and $\rho$ and $\theta$ are the coordinates in the log-polar space. The detailed operation process of inverse log-polar mapping can be divided into the following two steps: (i) entire CARTESIAN ladar range image is separated into a series of trapezoid-like quadrangle according to Eqs. (1)–(6), just as shaded portion shown in Fig. 3(b) (ii) every pixel of CARTESIAN ladar range image inside shaded portion is given a same range value, which is equal to the range value of unique log-polar sampling point inside shaded portion. Fig. 3(d) shows a CARTESIAN ladar range image which is converted from Fig. 3(c).

2.3. Image processing

Range image collected by ladar always contains background and various noises. Therefore, before the shape features of range image are extracted, an image processing must be employed to eliminate background and noises. In this paper, the method of fitting plane is used to eliminate ground and partial noises. Then, we extract the region of target object by using binary image. Finally, the method of extraction of connected components is used to clean the rest noise spots.

In principle, the mathematical expression of plane in 3D space can be written as $z = ax + by + c$. The plane and $z$ axis intersects at $(0, 0, c)$. In order to eliminate ground and partial noises in range image, the outermost ring range data of range image is used to fit a plane function using least square method, so that the optimal values of $a$, $b$, and $c$ can be obtained. Then, the ground of the range image and partial noises can be eliminated by giving up all the range data between plane $z=ax+by+(c-\Delta c)$ and plane $z=ax+by+(c+\Delta c)$. The value of $\Delta c$ should be properly selected, and this value should not be as big as possible. When this value is too big, too much useful shape information will be given up. However, if this value is too small, the ground and noises in range image will be not eliminated sufficiently. The selection of $\Delta c$ mainly depends on repeated experiments and level of noise. Fig. 4 (a) and (c) show Cartesian ladar range image and CARTESIAN ladar range image respectively which have been processed using above procedures.

![Fig. 4. Image processing. (a) Cartesian range image without background (b) Cartesian binary image (c) CARTESIAN range image without background (d) CARTESIAN binary image.](image-url)
method. Then, binary operation is used to extract the rest noise spots and region of target object.

When there are strong noises in the range image, the above operation is not enough to clean all the noises. As shown in Fig. 4 (a) and (c), many noise spots still exist, so the method of extraction of connected components is used to clean the rest noise spots. Extraction of connected components is a kind of morphological operator. The schematic diagram of extraction of connected components is shown in Fig. 5. Let $Y$ represents a connected component contained in a set $A$ and assume that a point $p$ of $Y$ is known. Then the following iterative expression yields all the elements of $Y$

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots$$

(8)

where, $B$ is a suitable structuring element, and $\oplus$ is a dilation operation in the field of imaging processing. The dilation operation can be explained using Eq. (9). With $X_{k-1}$ and $B$ as sets in $A$, the dilation of $X_{k-1}$ by $B$, denoted $X_{k-1} \oplus B$, is defined as

$$X_{k-1} \oplus B = \{2\left[(B)_z \cap X_{k-1}\right] \subseteq X_{k-1}\}.$$  

(9)

where, $(B)_z$ is the reflection of set $B$, and $(B)_z$ is the translation of set $B$ by point $z=(z_1, z_2)$, and more detailed explanation about the above operation can be found in [31]. In the first iterative step of Eq. (8), $X_0=p$. If $X_k=X_{k-1}$, the algorithm has converged and we let $Y=X_k$. In the last, all the connected components are found, and each connected component is labeled according to the size of connected component. Then, the biggest connected component is remained, and other connected components are abandoned. Fig. 4 (b) and (d) show the final Cartesian binary image and CARTESIAN binary image which have been processed using above two steps.

2.4. Combined moments(CM)

Moments have been widely and effectively used in the field of pattern recognition [27,28]. In general, lower order moments reflect the overall characteristics of the images, and higher order moments reflect the images detail [32]. Hu moments [33] is one of the most popular moments for its low computing cost and good properties of rotation, translation and scaling invariance. However, Hu moments don’t cover affine invariance, and affine transform can be used to approximate the projective transformation [34]. In addition, projective transform is an exact model of photographing a planar scene by ladar whose optical axis is not perpendicular to the scene. Therefore, three affine moments [34] also be chosen as shape descriptor. All the moments can be written as:

$$\phi_1 = \eta_{02} + \eta_{20}.$$  

(10)
\[ \phi_2 = (\eta_{30} - \eta_{12})^2 + 4\eta_{12}^2, \]
\[ \phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} + \eta_{03})^2, \]
\[ \phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2, \]

where, \( f \) is the transfer function, \( x \) is input vector, \( z \) is output vector, \( w \) denotes each layer weights, \( d \) and \( n_t \) denote the dimensions of input vector and hidden layer, respectively. In order to make the final output result of neural network fit the expected output result, the neural network needs to be trained using various learning algorithms. Backpropagation algorithm is one of the most successful training algorithms.

### 3. Experiments

In order to fully analyze the effect of log-polar sampling pattern on recognition performance of whole system, several comparative experiments using log-polar and Cartesian ladar range images were organized, and recognition rate was selected as evaluation indicator of recognition performance. In recent papers, recognition rate is the most commonly used performance evaluation criterion [11]. If the test object A in the scene is properly recognized as model object B by using recognition algorithm, then this condition is termed as successful recognition. The recognition rate refers to the number of successful recognitions divided by the total number of recognition tests. If the test object A in the scene is improperly recognized as model object B by using recognition algorithm, then this condition is termed as fail recognition. The misjudgment rate refers to the number of fail recognitions divided by the total number of recognition tests. For a good recognition algorithm, its recognition rate should be as big as possible, but its misjudgment rate should be as small as possible. The sum of recognition rate and all the misjudgment rates is 1.

Our research team has finished the log-polar imaging experiment using MOEMS mirror in [4]. However, it is still very difficult to get real range images of targets that meet all the requirements of a complex experiment. Thus, in this paper, five simulated objects with similar shape which come from the Princeton Shape Benchmark [36] were used to conduct a series of experiments. The five model objects are shown in Fig. 6(a). According to the basic algorithm process shown in Fig. 1, we firstly needed to collect large number of radar range images which were used as training samples set of neural network. In order to analyze the effect of different kinds of training samples on recognition performance, log-polar and Cartesian ladar range images were sampled at the same time. And the image resolution of Cartesian ladar range images was set to 128. Both the number of rings of log-polar sampling pattern \( M \) and the number of sampling points in each ring \( N \) were set to 128. Furthermore, in the process of collecting range images, each target object approximately possesses 36% of area of whole FOV. In other words, each target object possesses approximate 60% of length in the vertical and horizontal direction, and the target object is located at the center of FOV. In order to sample range image at scheduled attitude angle, as it is shown in Fig. 6(b), values of azimuth angle \( \theta \) were set to \(-180^\circ\), \(-175^\circ\), \(-170^\circ\), \(-165^\circ\), \(-160^\circ\), \(-155^\circ\), \(-150^\circ\), \(-145^\circ\), \(-140^\circ\), \(-135^\circ\), \(-130^\circ\), \(-125^\circ\), \(-120^\circ\), \(-115^\circ\), \(-110^\circ\), \(-105^\circ\), \(-100^\circ\), \(-95^\circ\), \(-90^\circ\), \(-85^\circ\), \(-80^\circ\), \(-75^\circ\), \(-70^\circ\), \(-65^\circ\), \(-60^\circ\), \(-55^\circ\), \(-50^\circ\), \(-45^\circ\), \(-40^\circ\), \(-35^\circ\), \(-30^\circ\), \(-25^\circ\), \(-20^\circ\), \(-15^\circ\), \(-10^\circ\), \(-5^\circ\), \(-0^\circ\), \(5^\circ\), \(10^\circ\), \(15^\circ\), \(20^\circ\), \(25^\circ\), \(30^\circ\), \(35^\circ\), \(40^\circ\), \(45^\circ\), \(50^\circ\), \(55^\circ\), \(60^\circ\), \(65^\circ\), \(70^\circ\), \(75^\circ\), \(80^\circ\), \(85^\circ\), \(90^\circ\), \(95^\circ\), \(100^\circ\), \(105^\circ\), \(110^\circ\), \(115^\circ\), \(120^\circ\), \(125^\circ\), \(130^\circ\), \(135^\circ\), \(140^\circ\), \(145^\circ\), \(150^\circ\), \(155^\circ\), \(160^\circ\), \(165^\circ\), \(170^\circ\), \(175^\circ\), \(180^\circ\).

In this case, for every target object, for every sampling pattern, \( q \) was set to 0, 5, 10, \ldots, 60°. In this way, one piece of range image possesses 865 range images with different attitude angles, \( q \). Considering the computation time and storage space required, the values of view angle \( q \) were set to 0°, 5°, 10°, \ldots, 60°. It is clear that \( 865 \times 1 \) range images with different attitude angles were obtained, among which there was only one piece of range image when the value of \( q \) was set to 0°.

Then, after getting training samples sets of log-polar range images and Cartesian range images, two BPNNs were established, and trained using the above two training samples sets, respectively. For each BPNN the number of input layer nodes corresponding to the CM was 10, and the number of output layer nodes corresponding to five model objects was 5. The number of neurons in the second layer was set to 100. For transfer functions, the first layer was 'Tansig' function and the second layer was 'Purelin' function. The network iterated 500 epochs to obtain a good result.
The Bayesian Regularization Backpropagation was chosen as training algorithm. In every output vector, the maximum value was changed to one, and the rest values were changed to zero. Therefore, we could obtain two BPNNs. The BPNNLP was trained using the training samples sets of log-polar range images, and the BPNNC was trained using the training samples sets of Cartesian range images. All the experiments were carried out in MATLAB ANN Library.

3.1. Log-polar CM and CARTESIAN CM

In the real application, ladar can directly provide log-polar range image. If CM vector can be directly computed by log-polar range image rather than CARTESIAN range image, it will avoid inverse log-polar mapping, and saves much running time. Whether the CM vector can be directly computed using log-polar range image is determined by checking whether the final CM vector can still keep rotation, translation and scaling invariance. Therefore, this experiment was carried out. As shown in Fig. 7, the first row is log-polar range images, and the second row is the corresponding

![Fig. 6. Ladar range image simulation. (a) model object (b) sampling model.](image)

![Fig. 7. Log-polar ladar range image and CARTESIAN ladar range images with different conditions.](image)
Table 1

<table>
<thead>
<tr>
<th>Combined moments</th>
<th>Similarity</th>
</tr>
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<tr>
<td>$\phi_1$</td>
<td></td>
</tr>
<tr>
<td>4.325</td>
<td>8.650</td>
</tr>
<tr>
<td>4.322</td>
<td>8.645</td>
</tr>
<tr>
<td>3.132</td>
<td>6.265</td>
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<tr>
<td>0.698</td>
<td>1.886</td>
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<tr>
<td>0.698</td>
<td>1.886</td>
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<tr>
<td>0.697</td>
<td>1.884</td>
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<tr>
<td>0.701</td>
<td>1.896</td>
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<td></td>
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</tbody>
</table>
| Note: ILPI, RLPI, TLPI and SLPI are the abbreviation of initial, rotational, translational and scaled log-polar image. ICI, RCI, TCI and SCI are the abbreviation of initial, rotational, translational and scaled CARTESIAN image.

3.2. Effect of FOV on recognition performance

In this section, a comparative experiment which contains three tests was carried out. In the first test, Cartesian ladar range images were selected as test images, and BPNNC was selected as recognition network. Its results are displayed using black curve. In the second test, log-polar ladar range images were selected as test images, and BPNNC was selected as recognition network. Its results are displayed using red curve. In the third test, log-polar ladar range images were selected as test images, and BPNNC was selected as recognition network. Its results are displayed using blue curve. In each test, when computing each final recognition rate in Fig. 8, since test images come from five different test objects and every recognition network contains shape information of these five test objects, there are five groups of recognition result corresponding to these five test objects, and every group contains one recognition rate and four misjudgment rates. The misjudgment rate appears when a test object is improperly recognized as other object by recognition algorithm. In other words, there are total five recognition rates and 20 misjudgment rates corresponding to these five test objects under every test condition. The mean value of these five recognition rates is used as final recognition rate under every test condition in Fig. 8. The mean value of the rest 20 misjudgment rates is used as final misjudgment rate. Since the sum of the final recognition rate and final misjudgment rate is 1, there is no need to show misjudgment rate in result figure. Furthermore, every group of recognition result was obtained by organizing 500 recognition tests. Therefore, every final recognition rate in Fig. 8 was obtained by organizing 2500 recognition tests. All the test images corresponding to every test object were produced using random azimuth, view and spin angle, and the largest view angle was still set to 60°. And the target object was still located in the center of FOV. Besides, the resolution of Cartesian test image was set to 128, and both the number of rings of log-polar test image and the number of sampling points in each ring were set to 128. When amplification rate (AP) of FOV is equal to 1, each target object in test image approximately possesses 36% of area of whole FOV. But when AP is equal to 0.5, each target object in test image approximately possesses 36/16 percent of area of whole FOV. Before using log-polar test image, it must be transformed into CARTESIAN test image. The reconstruction resolution (RR) of CARTESIAN test image was set to 256, 512, 768 and 1024 respectively which correspond to Fig. 8(a), (b), (c) and (d). Every recognition rate was obtained by organizing 500 recognition tests.

In order to research the effect of AR on recognition performance, the experiment in this section was organized, and the final results are shown in Fig. 8. In this figure we can find that all

![Fig. 8. Recognition rate as a function of FOV. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)](image-url)
recognition rates decline as the AR increases no matter sampling pattern, recognition network and RR. However, the decline tendency is dramatical for Cartesian sampling pattern, and the decline tendency is relatively smooth for log-polar sampling pattern. When the AR is equal to 10, the recognition rate approximately reaches up to 90% for log-polar sampling pattern in Fig. 8(b) and (d). However, the recognition rate just approximates 25% for Cartesian sampling pattern. Furthermore, the difference between two recognition networks for log-polar sampling pattern is not obvious, but the BPNNC has a slight advantage for entire curve. We can also find that the decline speed of log-polar sampling pattern is slower in Fig. 8(b) and (d) than that of the rest two figures, but the difference in this two figures is very small. Moreover, there are two strange findings in Fig. 8. Firstly, the recognition rate of Cartesian image has a slight increase as the AR exceeds 18. In theory, we believe that the recognition rate will continuously decline as AR increase until its theoretical minimum. The first row in Fig. 9 explains its reason. When AR exceeds 12, the target object becomes a straight line since the severe lack of sampling points. As the AR continuously increases, the straight line become shorter step and step, and its shape become more similar to initial object. This is why the recognition rate has a slight increase. The final recognition rate is equal to 20% since there are just five objects in this experiment. Secondly, compared to the result in Fig. 8(b), there is worse recognition performance in Fig. 8(c) for log-polar image. In theory, we believe that the recognition rate will continuously decline as AR increase. The last three rows in Fig. 9 successfully explain its reason. The existence of blind hole of log-polar sampling pattern causes this phenomenon. In the second and third row in Fig. 9, it is very easy to find this blind hole when the RR exceeds 768. We can also find that the blind hole become smaller with the increasing of RR. The last row in Fig. 9 visually explains its reason. When the RR is small, the distance between two adjacent CARTESIAN pixels is greater than the size of blind hole, so the blind hole is invisible. But as the RR continually increases, the distance between two adjacent CARTESIAN pixels begins to be less than the size of blind hole, so the blind hole begins to be visible. As shown in the last row in Fig. 9, the size of blind hole in CARTESIAN image will progressively become smaller with the increasing of RR until there are new pixels dropping into this blind hole, which conforms to the phenomenon observed in the second and the third row. So we can conclude that when target object is located in the center of FOV, recognition rate of log-polar sampling pattern is less sensitive to the changing of FOV than that of Cartesian sampling pattern. The BPNNC has a slight advantage compared to the BPNNLP in the aspect of recognition rate. The higher RR does not always mean the higher recognition rate.

3.3. Effect of object position on recognition performance

Log-polar sampling pattern is a kind of non-uniform sampling pattern. Obviously, target object position in log-polar ladar range image will influence recognition performance of whole system. In order to research the effect of object position on recognition performance, the experiment in this section was organized according to entirely same rule just as shown in Section 3.2. But in this experiment, the target object is not located in the center of FOV, and object position becomes an independent variable rather than a constant variable. When object position value is equal to 1, the centroid of target object is located in the center of FOV. When object position value is equal to 0, the centroid of target object is located in the edge of FOV. Therefore, the object position is distributed linearly from center to edge of FOV as object position value decreases from 1 to 0. Furthermore, in Fig. 10, the curve with solid symbol denotes that AR is equal to 3, and the curve with hollow symbol denotes that AR is equal to 6.

The final experiment results are shown in Fig. 10. We find that object position does not have any effect on recognition rate of Cartesian range image, and the recognition rate of AR=6 is less than that of AR=3 which conforms to the conclusions in Section 3.2. We can also find that recognition rate of log-polar image dramatically decreases as the object moves from center to edge of FOV, and the slope of recognition rate curve of AR=6 is steeper than that of AR=3. That means the recognition rate difference between AR=3 and AR=6 progressively increase as object moves from center to edge of FOV. When AR=3, the recognition rate difference between BPNNLP and BPNNC for log-polar image is very small, and the BPNNLP has a slight advantage. But when AR=6, the performance of BPNNLP is better in the edge part of FOV, and the result is reverse in the center part of FOV. All the corresponding curves just has a very small difference among Fig. 10(b), (c) and (d). That means when RR is not too small, RR does not have obvious effect on recognition performance. Moreover, we can also find that the recognition rate of log-polar image is better than that of Cartesian image when RR is small.
of Cartesian image in the center part of FOV, but the result is reverse in the edge part of FOV. Therefore, we can divide the FOV into two parts in the real application. The first part corresponding to center of FOV are used as recognition area, and the second part corresponding to edge of FOV are used as searching area, which balances the conflict between high recognition performance and large searching area. So we can conclude that as object position changes from center to edge of FOV, recognition rate of log-polar ladar range images will dramatically decline. However, object position does not have effect on recognition rate of Cartesian ladar range image. The larger AR, the steeper the recognition rate curve of log-polar image will be. The BPNNC has a slight advantage compared to the BPNNLP in the aspect of recognition rate. The higher RR does not always mean the higher recognition rate.

3.4. Effect of noise on recognition performance

In the real application, radar range image always contains various noises. In order to research the effect of noise on recognition performance, the experiment in this section was organized according to entirely same rule just as shown in Section 3.2. But in this experiment, all the test images were added Gaussian noise. The strength of noise was described by standard deviation. The mean value of Gaussian noise was 0. Furthermore, in Fig. 11, the curve with solid symbol denotes that AR is equal to 3, and the curve with hollow symbol denotes that AR is equal to 6.

The final experiment results are shown in Fig. 11. We can find that all recognition rates decline dramatically as noise strength increases no matter sampling pattern, recognition network, AR and RR. All the recognition rates drop to the theoretical minimum when noise strength is equal to 0.7. We can also find that log-polar range image has an obvious advantage than Cartesian range image under any condition with noise in the aspect of recognition rate. As the AR increases, all the recognition rates decrease, which conforms to the conclusions in Section 3.2. But the difference between the corresponding log-polar recognition rate curves with different AR progressively decreases as the RR increases. However, this difference is not obvious when the RR exceeds 512. Furthermore, we can find that the BPNNLP has a slight advantage compared to the BPNNC in the aspect of recognition rate, which is opposite to previous conclusion when range image never contains any noise. Therefore, we can conclude that recognition performance of log-polar ladar range images is less sensitive to noise than that of Cartesian ladar range images. The higher RR does not always mean the higher recognition rate. The BPNNLP has a slight advantage compared to the BPNNC in the aspect of recognition rate in this condition with noise.

3.5. Recognition experiment based on real range images

In order to fully analyze the effect of log-polar sampling pattern on recognition performance of whole system, recognition experiment based on real range images needs to be carried out. Although our research team has finished the log-polar imaging experiment using MOEMS mirror in [4], it is still very difficult to obtain a large number of real range images that meet all the requirements of a complex experiment. Therefore, two small experiments based on real range images were carried out in this section.

In the first experiment, the BPNNLP in the paper was used as recognition network, and real tank range images shown in Fig. 12 were used as test images. The shape of this tank is similar to that of the first tank in Fig. 6(a). Considering the difficulty of collecting a large number of real range images, the test images were sampled at a constant view angle and variable azimuth angles, and the azimuth angles \( \theta \) were set to \(-180^\circ, -170^\circ, -160^\circ, \ldots, 170^\circ\). In this case, 36 real log-polar tank range images with different azimuth angles were obtained. The final recognition result is 33-0-2-0-1. In other words, there are 33 real tank range images which are considered to be more similar to the range image of the first tank in Fig. 6(a). This experiment result conforms to the basic fact.

In the second experiment, the two objects in Fig. 12 were used as expected targets. The log-polar range images of these two
targets were also collected at a constant view angle and variable azimuth angles. In the last, 72 log-polar range images were obtained. All the log-polar range images were used as test images, and six new BPNN training sample sets were produced by selecting range image from every one, two, three, four, six, and twelve images, respectively. In other words, the first set contains 72 log-polar range images, and the last set just contains 6 log-polar range images. Therefore, six different BPNNs were obtained, and six recognition tests were carried out. The final recognition result is shown in Table 2. We can find the result is relatively good. Especially, for star object, even for a small size of training sample set, it can be also accurately recognized.

### 4. Conclusion

In this work, we establish a target recognition system which uses log-polar ladar range images as input data. This system consists of two processes, BPNN training and recognition. In this system, Hu moments and affine moments are used as global feature representation method, and BPNN is used as feature classifier. In order to fully analyze the effect of log-polar sampling pattern on recognition performance, several comparative experiments based on simulated and real range images are carried out. Eventually, several important conclusions are drawn: (i) in order to keep invariant properties of translation, rotation and scaling, all the moment vectors must be computed after log-polar ladar range images have been transformed into CARTESIAN ladar range images (ii) when object is located in the center of FOV, recognition rate of log-polar range images is less sensitive to the changing of FOV than that of Cartesian sampling pattern, and BPNNC has a slight advantage compared to BPNNLP in the aspect of recognition rate (iii) object position changes from center to edge of FOV, recognition rate of log-polar range images will dramatically decline, and it is not affected for Cartesian ladar range images, and BPNNC has a slight advantage compared to BPNNLP in the aspect of recognition rate (iv) recognition rate of log-polar range images is less sensitive to noise than that of Cartesian ladar range images, and BPNNLP has a slight advantage compared to BPNNC in the aspect of recognition rate when range image contains noises. Furthermore, all the experiments prove that when RR is not too small, the effect of RR on recognition performance is weak. Therefore, we suggest that FOV should be divided into two parts in the real application. The first part corresponding to center of FOV are used as recognition area, and the second part corresponding to edge of FOV are used as searching area, which balances the conflict between excellent recognition performance and large searching area. And BPNNLP is more suitable as feature classifier since the existence of noise.

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Appendix

Considering that there are many acronyms in this paper, although they have been explained when they first appear, they still cause a big trouble for reader to seek their meanings, so Table 3 is added in order to help reader seek each acronym’s meaning.

References