Robust Sliding Mode Control for Robots Driven by Compliant Actuators

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Abstract—Compliant actuators can offer many attractive features over stiff actuators in real human–robot interaction applications, such as low output impedance, smooth force transmission, and shock tolerance. This brief focuses on a robust sliding mode control (SMC) methodology for robotic systems with compliant actuators. First, a continuous SMC design is introduced due to its advantages of strong robustness and chattering attenuation. However, this continuous SMC structure cannot guarantee a high tracking performance in the presence of mismatched disturbances in compliantly actuated robots. Meanwhile, in many application fields, compliantly actuated robots are affected by different kinds of time-varying disturbances, including external environmental disturbances, internal parameter uncertainties, and frictions, which may be in the form of constant, ramp, and parabolic disturbances. To estimate such unknown disturbances, a generalized proportional integral observer (GPIO) technique is employed. By designing a new sliding surface with the help of disturbance estimation, a GPIO-based continuous SMC method is synthesized, which is used to deal with matched/mismatched time-varying disturbances. A detailed stability analysis of the closed-loop system is also presented. Experimental results under three different test conditions are provided to illustrate the promising tracking performance of the proposed control strategy.

Index Terms—Compliant robot, disturbance attenuation, mismatched disturbance, generalized proportional integral observer (GPIO), sliding mode control (SMC).

I. INTRODUCTION

IN RECENT years, due to the rapid population aging in most developed nations, the demand for robots in assistance, services, and rehabilitation has been increasing significantly. In those applications, robots have to direct interaction with humans, and safety is the most critical concern [1]. Generally, robots are always devised with stiff actuators for accurate and fast trajectory tracking, which results to potential safe threat to human in the human–robot interaction. This has become the main motivation for the investigation of compliant actuators.

Unlike rigid actuators, compliant actuators are developed by introducing elastic element between the load and the motor. This design structure makes compliant actuators obtained many attractive features, including low output impedance, smooth force transmission, shock tolerance, back driveability, and energy efficiency [2]. A detailed review on the comparison of different compliant actuators is found in [3]. However, due to the selection of spring stiffness, current compliant actuator designs face a common limitation in force control performance, back drivability, and intrinsic compliance [4]. To overcome the limitations of conventional compliant actuators, a new compliant actuator was proposed in [5]. The key novelty is the design of the combination of soft and hard springs, where two springs are utilized in the low-force and the high-force range of compliant actuators, respectively. Based on this two-stage stiffness, the compliant actuator offers low out impedance and high force fidelity, and simultaneously extends the bandwidth and force range. Therefore, the performance of compliant actuators is greatly improved.

This brief is target to devise a robust tracking control method for robots driven by compliant actuators. However, it is well known that a compliantly actuated robot system is nonlinear and the order of the system is twice higher than that of the rigid robot due to the coupling of the motor and link [6]–[9]. In addition, the control performance of such a system is always seriously suffered from unknown matched/mismatched time-varying disturbances, mainly including unknown parameter uncertainties, external load disturbances, and unmodeled dynamics [10]. These inherent system properties bring a very challenging task for control engineers to design a robust control law for compliantly actuated robots.

With the rapid progress of manufacturing technology in integrated circuit, computer software, and modern control theories, it is entirely feasible to design and implement a robust control law with advanced control methodologies to improve the control performance of mechanical systems, including compliantly actuated robots. For this reason, various nonlinear control schemes have been employed to compliantly actuated robots in the literature, such as backstepping control [11], [12], adaptive control [10], predictive control [13], sliding mode control (SMC) [14], disturbance rejection control [15], singular perturbation control [16], passivity-based control [17], and intelligent control [18]. The aforementioned control schemes...
have improved the control performance of compliantly actuated robots from different aspects. However, when robotic systems are perturbed by severe disturbances, which may include external load disturbances, internal parameter uncertainties, and unmodeled dynamics, most of the aforementioned control schemes will result in a degradation of the control performance, since conventional feedback control approaches usually suppress the disturbances in a passive way rather than react or cancel them actively. The notable exception to this case is SMC and disturbance rejection control, which are robust with respect to system disturbances [19], [20].

SMC has been applied to compliant robotic systems since its strong robustness to unknown exogenous disturbances, parameter variations, and model perturbations [14]. However, in order to guarantee satisfactory performance, a discontinuous control action is usually adopted in the traditional SMC, which causes a well-known chattering phenomenon and probably damages the actuator of robotic control systems [21]. It should also be pointed out that the conventional SMC approach cannot effectively handle mismatched disturbances that are not in the same channel with the control input [22]. Disturbance rejection control has been employed in compliant robotic systems [15], [23] and also other systems [24]–[26] to improve their robustness. Again this control method is only concentrated on matched disturbances compensation [27]–[29]. Hence, the most directed approach to obtain a better control performance is to synthesize a robust controller using the SMC and disturbance rejection control technique.

In this brief, a general trajectory tracking control scheme is developed for compliantly actuated robots subjected to unknown matched/mismatched time-varying disturbances. First, a continuous SMC design methodology developed in [30]–[32] is introduced here due to its advantages of strong robustness and chattering attenuation. However, it is shown that the high tracking performance of the closed-loop system cannot be obtained under this traditional SMC structure since mismatched disturbances acting on different channels from the control input in compliantly actuated robots. Considering the fact that compliantly actuated robots are generally affected by time-varying disturbances, including external environmental disturbances, internal parameter uncertainties, and frictions, which may be in the form of constant, ramp, and parabolic disturbances, a generalized proportional integral observer (GPIO) technique is employed to estimate such unknown disturbances. With the help of two GPIOs, a new sliding surface is designed for unknown matched/mismatched time-varying disturbances rejection problem of the compliantly actuated robots (2), such that system output $q$ tracks its desired trajectory $q_r$ as fast and accurate as possible.

## III. Problem of the Conventional SMC Design

In this section, the following two steps are given. First, due to its advantages of strong robustness and chattering attenuation, a continuous SMC scheme developed in [30]–[32] is introduced for the tracking control of compliantly robotic systems (2). Then, the existing problem of this conventional SMC design philosophy is listed in detail.

Let $q_i(t) = [q_{r1}(t), q_{r2}(t), \ldots, q_{rn}(t)]^T \in \mathbb{R}^n$ represent the given signal of reference positions, where $q_r(t), \dot{q}_r(t), \ddot{q}_r(t)$,
As follows:

\[ a(t) = e_4 + c_3 e_3 + c_2 e_2 + c_1 e_1 \]

where \( e_i = q_i^{(i-1)} - x_i \), \( c_i = \text{diag}(c_{i1}, c_{i2}, \ldots, c_{in}) \), \( c_{i1}, c_{i2}, \ldots, c_{in} > 0 \), is constant, which can be obtained, such that the roots of the following characteristic polynomial \( p_i(s) \) in the complex variable \( s \):

\[ p_i(s) = s^4 + c_i s^3 + c_i s^2 + c_i s + c_i \]

are all in the left-hand side of the complex plane.

The continuous SMC law is designed as

\[ u = J_0(q_4(t) + c_4 e_4 + c_3 e_3 + c_2 e_2 + c_1 e_1 + \kappa \int_0^t \text{sgn}(\sigma)d\zeta) \]

where \( \kappa = \text{diag}(\kappa_1, \kappa_2, \ldots, \kappa_n) \), \( \kappa_1, \kappa_2, \ldots, \kappa_n > 0 \). Combining (3) and (5) gives

\[ \dot{\sigma} = -\kappa \text{sgn}(\sigma) - \dot{d}_2(t). \]

It can be shown from (6) that the tracking error \( e_1 \) of system (2) will be driven to sliding-mode surface \( \sigma = 0 \) by the control law (5) in finite time as long as control gain \( \kappa \) is designed, such that \( \kappa_i > \sup_{t > 0} |\dot{d}_2(t)| \), \( i = 1, 2, \ldots, n \), where \( \dot{d}_2(t) \in d_2(t), d_2(t) = [d_{21}(t), d_{22}(t), \ldots, d_{2n}(t)] \) is in \( \mathbb{R}^n \). When \( \sigma = 0 \), the sliding motion is obtained as follows:

\[ e_1^{(1)} + c_1 e_1^{(3)} + c_3 e_1^{(3)} + c_2 e_1^{(3)} + c_1 e_1 = -c_1 d_1 + c_3 d_1. \]

Remark 1: The sliding mode variable \( \sigma \) in control law (5) is not available, because the signal \( \dot{e}_\alpha \) in \( \dot{e}_\alpha \) could not be measured directly. For calculating the sign of sliding variable \( \sigma \) in (5), a function \( g(t) \) is defined [28], \( g(t) = \int_0^t \sigma dt = e_4(t) - e_4(0) + \int_0^t e_4 e_3(t) + c_3 e_3(t) + c_2 e_2(t) + c_1 e_1(t) dt \), and then, \( \text{sgn}(\sigma) \) can be obtained, \( \text{sgn}(\sigma) = \text{sgn}(g(t) - g(t - i)) \), where \( i \) is the fundamental sampling time and \( \text{sgn}(\sigma) = \lim_{t \to 0} [g(t) - g(t - i)]/i \).

Remark 2: Equation (7) implies that the trajectory tracking error cannot be driven to the desired equilibrium point even the control law (5) can force the tracking error of compliantly actuated robots (2) to reach the sliding surface in finite time because of nonzero term \( c_1 d_1 + c_3 d_1 \). That is to say the continuous SMC scheme, including conventional SMC, is only insensitive to matched uncertainties \( d_2 \) but sensitive to mismatched uncertainties \( d_1 \).

IV. PROPOSED ROBUST SMC DESIGN

In this brief, we target to design a general trajectory tracking control framework for robots driven by compliant actuators subjected to unknown matched/mismatched time-varying disturbances. A robust continuous SMC method is proposed by the following two steps. Two GPISOs are first employed to estimate unknown matched/mismatched time-varying disturbances, respectively. Then, under the assistance of disturbance estimation, a GPIO-based continuous SMC approach is proposed for compliantly actuated robots in order to implement online matched/mismatched time-varying disturbances’ compensation.

A. Controller Design

The objective of the following is to design two GPIOs for compliantly robotic system (2), which can estimate time-varying disturbances acting on the motor/link side, respectively. This disturbance estimation technique is based on the differential flatness property, which was proposed and developed in [35] and [36].

It is assumed that the first \( m \) and \( p \) time derivatives of \( d_1 \) and \( d_2 \) exist, respectively, and \( d_1 \) and \( d_2 \) can be, respectively, expressed by the following Taylor polynomials [37]:

\[ d_1 = \sum_{i=0}^{m-1} a_i t^i, \quad d_2 = \sum_{j=0}^{p-1} y_j t^j \]

where \( a_i = \text{diag}(a_{i1}, a_{i2}, \ldots, a_{in}) \), \( a_{i1}, a_{i2}, \ldots, a_{in} > 0 \) and \( y_j = \text{diag}(y_{j1}, y_{j2}, \ldots, y_{jn}) \), \( y_{j1}, y_{j2}, \ldots, y_{jn} > 0 \) are unknown constants.

Remark 3: As shown in (8), the unknown time-varying disturbances \( d_1 \) and \( d_2 \) can be modeled as the Taylor polynomial form with fixed \( m-1 \), \( p-1 \) degree, respectively. According to the nature of disturbances, we can choose the order \( m \) and \( p \). For example, the disturbances in the form of constant, ramp, and parabolic and their combinations can be expressed as (8) by setting the order \( m = 1, 2 \), and 3, respectively. Generally, in order to obtain the higher observer accuracy, the larger order \( m \) should be selected. However, this treatment will increase the computational burden obviously. Hence, a tradeoff should be taken into account between the observer accuracy and the computational burden in practice [38].

Let \( z_0 = d_1, z_1 = d_1, \ldots, z_{m-1} = d_1^{(m-1)}, y_0 = d_2, y_1 = d_2, \ldots, y_{p-1} = d_2^{(p-1)} \), two GPIOs, which are used to estimate unknown time-varying disturbances \( d_1 \) and \( d_2 \), respectively, are devised for system (2) [38], [39]

\[ \begin{aligned}
\dot{x}_1 &= \dot{x}_2 + \lambda_{1,m-2} (x_1 - \hat{x}_1) \\
\dot{x}_2 &= \dot{z}_1 + \lambda_{1,m-1} (x_1 - \hat{x}_1) \\
\dot{z}_1 &= \lambda_{1,1} (x_1 - \hat{x}_1) \\
\hat{x}_1 &= 0, 1, \ldots, m - 2 \\
\hat{x}_{m-1} &= \lambda_{1,1} (x_1 - \hat{x}_1) \\
\dot{x}_3 &= \hat{x}_0 + J_0^{-1} r + \lambda_{2,p+1} (x_3 - \hat{x}_3) \\
\hat{x}_0 &= \lambda_{2,1} (x_3 - \hat{x}_3) \\
\dot{x}_j &= \lambda_{2,1} (x_3 - \hat{x}_3) \\
\hat{x}_{p-1} &= \lambda_{2,1} (x_3 - \hat{x}_3)
\end{aligned} \]

\[ \begin{aligned}
\dot{\lambda}_{i,j} &= \lambda_{i,j} (x_3 - \hat{x}_3) \\
\hat{\lambda}_{i,j} &= \lambda_{i,j} (x_3 - \hat{x}_3)
\end{aligned} \]

where \( x_1 \) and \( x_3 \) can be obtained from the outputs of system (2), \( \hat{x}_1, \hat{x}_2, \hat{x}_0, \hat{x}_1, \ldots, \hat{x}_{m-1} \) are the estimations of \( x_1, x_2, z_0, z_1, \ldots, z_{m-1} \), respectively, \( \hat{x}_3, \hat{x}_4, \hat{\theta}_0, \hat{\theta}_1, \ldots, \hat{\theta}_{p-1} \) are the estimations of \( x_3, x_4, y_0, y_1, \ldots, y_{p-1} \), respectively, and \( \lambda_{1,1} = \text{diag}(\lambda_{1,1}^{(1)}, \lambda_{1,1}^{(2)}, \ldots, \lambda_{1,1}^{(m-1)}), \lambda_{2,1}^{(1)}, \lambda_{2,1}^{(2)}, \ldots, \lambda_{2,1}^{(p-1)} > 0 \), \( i = 1, 2, \ldots, m + 2 \), and \( \lambda_{2,1} = \text{diag}(\lambda_{2,1}^{(1)}, \lambda_{2,1}^{(2)}, \ldots, \lambda_{2,1}^{(p-1)}) \), \( \lambda_{2,1}^{(1)}, \lambda_{2,1}^{(2)}, \ldots, \lambda_{2,1}^{(p-1)} > 0 \) are the
coefficients of GPIO I and II, respectively, which are chosen so that the roots of the characteristic polynomials $p_{o1}(s) = [p_{o1}^1(s), p_{o1}^2(s), \ldots, p_{o1}^n(s)]^T$ and $p_{o2}(s) = [p_{o2}^1(s), p_{o2}^2(s), \ldots, p_{o2}^n(s)]^T$ in the complex variable $s$

\[
p_{o1}(s) = s^{m+2} + \lambda_{1,m+2}s^{m+1} + \lambda_{1,m+1}s^m + \cdots + \lambda_{1,2}s + \lambda_{1,1},
\]

\[
p_{o2}(s) = s^{p+2} + \lambda_{2,p+2}s^{p+1} + \lambda_{2,p+1}s^p + \cdots + \lambda_{2,2}s + \lambda_{2,1}.
\]

are located in the left-hand side of the complex plane.

With the assistance of GPIOs, the estimations of unknown matched/mismatched time-varying disturbances are obtained. A new sliding surface for a state-space model (2) with such mismatched disturbance estimation is given by

\[
\sigma = \dot{e}_4 + c_4e_4 + c_3e_3 + c_2e_2 + c_1e_1
\]

(12)

where $e_1 = q_r - x_1$, $e_2 = \dot{q}_r - x_2$, $e_3 = \ddot{q}_r - (x_3 + \dot{z}_0)$, $e_4 = q_r^{(3)} - (x_4 + \dot{z}_1)$, and $c_1 = \text{diag}(c_{1,1}, c_{1,2}, \ldots, c_{1,n})$, $c_{1,1}, c_{1,2}, \ldots, c_{1,n} > 0$, is control parameter.

The proposed GPIO-based continuous SMC law is designed as

\[
u = \int_0^t \left( q_r^{(4)} + c_4e_4 + c_3e_3 + c_2e_2 + c_1e_1 - \eta_0 - \dot{\eta}_0 - \ddot{\eta}_2 \right)
\]

\[+ \kappa \int_0^t \text{sgn}(\sigma) d\zeta \right).
\]

The control structure of compliantly actuated robots under the GPIO-based continuous SMC design methodology is shown in Fig. 1.

**B. Stability Analysis**

**Lemma 1** [40]: Consider $x_i \in \mathbb{R}$, $i = 1, 2, \ldots, n$, $0 < p \leq 1$ is a positive real number, the following inequality is established:

\[
(|x_1| + |x_2| + \cdots + |x_n|)^p \leq |x_1|^p + |x_2|^p + \cdots + |x_n|^p \leq n^{1-p}(|x_1| + |x_2| + \cdots + |x_n|)^p.
\]

**Lemma 2** [41]: Consider a nonlinear system $\dot{x} = F(x, \varphi)$ which is input-to-state stable (ISS). If the input satisfies $\lim_{t \to \infty} \sigma(t) = 0$, then the state $\lim_{t \to \infty} x(t) = 0$.

**Theorem 1**: Considering compliantly actuated robots (2) under the matched/mismatched time-varying disturbances, a robust SMC scheme in the form of (9), (10), (12), and (13) is given. If the gain of the control law (13) satisfies $\kappa_i > e_i$, the tracking error of system (2) will converge to the desired equilibrium point asymptotically.

**Proof**: According to (13), the sliding surface (12) can be rewritten as follows:

\[
\sigma = \dot{e}_4 + c_4e_4 + c_3e_3 + c_2e_2 + c_1e_1
\]

\[= q_r^{(4)} - J_0^{-1} \tau - \dot{z}_2 + c_4e_4 + c_3e_3 + c_2e_2 + c_1e_1
\]

\[= -\kappa \int_0^t \text{sgn}(\sigma) d\zeta - \eta_0.
\]

(14)

Consider the following Lyapunov function $V(\sigma) = (1/2)\sigma^T \sigma$.

Taking the derivative of sliding surface (14) yields

\[
\dot{V}(\sigma) = \sigma^T \dot{\sigma} = -\sum_{i=1}^n (k_i |\sigma_i|) - \sum_{i=1}^n (\phi_i |\sigma_i|)
\]

\[= -\sum_{i=1}^n (k_i - |\phi_i|) |\sigma_i| \leq \sum_{i=1}^n (k_i - e_i) |\sigma_i| \leq -\sum_{i=1}^n (k_i - e_i) |\sigma_i| \leq -L \sum_{i=1}^n |\sigma_i| \leq -\sqrt{2L} \sum_{i=1}^n V_i^T(\sigma_i)
\]

(17)

where $L = \min(k_i - e_i)$, $|\phi_i(t)| \leq e_i$, $e_i > 0$, and $\phi_i(t) \in \phi = [\phi_1, \phi_2, \ldots, \phi_n]^T \in \mathbb{R}^n$. Define $V_i(\sigma_i) = (1/2)\sigma_i^T$. $i = 1, 2, \ldots, n$, yields

\[
V(\sigma) = (1/2)\sigma^T \sigma = \sum_{i=1}^n V_i(\sigma_i)
\]

(18)

Combining Lemma 1 gives

\[
\dot{V}(\sigma) = -\sqrt{2L} \sum_{i=1}^n V_i^T(\sigma_i)
\]

\[\leq -\sqrt{2L} \left( \sum_{i=1}^n V_i(\sigma_i) \right)^{1/2} = -\sqrt{2LV}(\sigma).
\]

(19)

If $k_i > e_i$, the defined errors $e_i$ arrive at sliding surface $\sigma = 0$ (12) in finite time. The sliding motion is then described as

\[
\sigma = \dot{e}_4 + c_4e_4 + c_3e_3 + c_2e_2 + c_1e_1
\]

\[= (q_r^{(4)} - (x_4 + \dot{z}_1)) + c_4(q_r^{(3)} - (x_4 + \dot{z}_1))
\]

\[+ c_3(q_r - (x_3 + \dot{z}_0)) + c_2(q_r - x_2) + c_1(q_r - x_1)
\]

\[= 0.
\]

(20)
be verified that the following system:

\[ e_1^{(4)} + c_3 e_1^{(3)} + c_2 e_1 e_1 + c_1 e_1 = - (e_{z_2} + c_4 e_{z_1} + c_3 e_{z_2}) \]

(21)

is exponentially stable. With this result, it can be derived from [38, Lemma 5.5] that system (21) is ISS. According to the GPIO dynamics, the disturbance estimation \( \hat{z}_i(t) \) of GPIO (9) can track the disturbance \( z_i(t) \) asymptotically, i.e., \( \lim_{t \to \infty} e_i(t) = \lim_{t \to \infty} (z_i(t) - \hat{z}_i(t)) = 0 \). Then, it can be derived from Lemma 2 that the states of system (21) satisfy \( \lim_{t \to \infty} e_i(t) = \lim_{t \to \infty} (q_i(t) - x_i(t)) = 0 \). This implies that the tracking error will slide to the equilibrium point asymptotically under the proposed control law. This completes the proof.

Remark 4: According to the aforementioned stability analysis, it is noted that the tracking error of compliantly actuated robots (2) is first driven to the sliding surface (12) in finite time. Then, it converges to the desired equilibrium point in an asymptotically exponential manner along the sliding surface (12), since the offset caused by \( d_1, d_2, d_3 \) and \( \hat{d}_1 \) can be removed from the sliding surface by utilizing the estimations \( \hat{z}_o, \hat{z}_1, \) and \( \hat{z}_2 \). Meanwhile, an estimation \( \hat{y}_0 \) is introduced to the control law (13) for compensating the effect of matched time-varying disturbances on system (2). This is the main reason why the proposed robust control scheme is insensitive to matched/mismatched time-varying disturbances simultaneously.

V. EXPERIMENTAL RESULTS

A cable-driven version of the compliant actuator in [42], which is mainly consists of a servomotor with a rotary encoder, a set of linear springs, a ball screw, and two potentiometers, is illustrated in Fig. 2. The motion from motor converts the rotary motion of the shaft into linear motion of the ball screw nut via a coupler. Then, the motion of nut is transmitted to the output carriage via the linear springs, while the carriage drives a robot joint though a pair of cables. The encoder installed on the motor is used to measure the angular displacement of the motor and the ball screw, the linear potentiometer is used to measure the displacement of the linear spring, and another rotary potentiometer installed in the robot joint is used to measure the joint angle. In this section, the proposed control methodology is validated by experimental studies. Two comparative control strategies, including continuous SMC and continuous SMC + GPIO, are carried out here on a single-link compliantly actuated robot, respectively. In this prototype, the length and mass of the link is 0.45 m and 0.20 kg, respectively, the nominal parameter of motor inertia \( J_0 \) used in the experiment is selected as \( J_0 = 2.2 \times 10^{-6} \) (kg · m²). The control schemes are implemented on a dSPACE DS1007 real-time hardware kit. The sampling periods of two control methods are both set to be 0.1 ms.

In this particular case, the controller parameters in the continuous SMC controller (5) and the continuous SMC + GPIO controller (13) are set to be of the following fourth-order polynomial form:

\[ p_c(s) = (s^2 + 2 z_c \omega_c s + \omega_c^2)^2 \]

with

\[ [c_{11}, c_{21}, c_{31}, c_{41}] = [\omega_c^4, 4 z_c \omega_c^3, 2 \omega_c^2 + 4 z_c \omega_c^2, 4 z_c \omega_c] \]

where \( \omega_c = 100 \) and \( z_c = 1 \). The corresponding observer gains in GPIO I (9) and GPIO II (10) are chosen in accordance with the following fifth-order polynomial form:

\[ p_o(s) = (s + k_o)(s^2 + 2 z_o \omega_o s + \omega_o^2)^2 \]

with

\[ \left[ \lambda_{10}^1, \lambda_{11}^1, \lambda_{12}^1, \lambda_{13}^1, \lambda_{14}^1 \right] = \left[ \lambda_{10}^2, \lambda_{11}^2, \lambda_{12}^2, \lambda_{13}^2, \lambda_{14}^2 \right] = [\omega_o^2 k_o, 4 z_o \omega_o k_o + \omega_o^2, 4 z_o \omega_o^3 + 2 \omega_o^2 k_o + 4 z_o^2 \omega_o^2 k_o, 2 \omega_o^2 + 4 z_o^2 \omega_o^2 + 4 z_o \omega_o k_o + 5 z_o^2 \omega_o] \]

where \( k_o = a_0 = 500 \) and \( z_0 = 1 \). The following three experiments are implemented to validate the robustness of the proposed continuous SMC + GPIO approach against mismatched/matched time-varying disturbances.

A. Test 1: GPIO-Based Continuous SMC Design Strategy Versus Continuous SMC Design Strategy

In the first experiment, we use experimental results to compare the control performance between the proposed continuous SMC + GPIO control law (13) and the continuous SMC control law (5). Fig. 3 shows the tracking curves of link position and control torque (or control input) under two controllers. As can be seen in Fig. 3(a), when the continuous SMC approach is employed, the position of robot could not converge to its desired trajectory. According to (7), we know that the continuous SMC scheme is sensitive to the mismatched disturbance \( d_{11} \) and this mismatched disturbance is always nonzero even though external disturbance is unloaded. From Fig. 3(c), we observe that the robotic output \( q_1 \) tracks its...
desired trajectory $q_{r1}$ quickly. Tracking error in steady state is acceptable in the interval $[-0.026, 0.022]$. The reason is that mismatched disturbance estimation $d_{11}$ and its derivatives can be obtained with the aid of GPIO I, and simultaneously applied to continuous SMC controller for disturbance compensation, and then, the sliding motion along the sliding surface can drive link position to its commanded trajectory as accurate as possible. The results of previous tests show that the robot under the continuous SMC control strategy is not able to track
B. Test 2: GPIO-Based Continuous SMC Design Strategy Under Mismatched Time-Varying Disturbances

In the second experiment, the robustness against mismatched time-varying disturbances applied to compliantly actuated robot is investigated using the continuous SMC + GPIO controller. Corresponding reference trajectory is sine wave (amplitude is 18° and frequency is 1 Hz). The trajectories tracking curves of link position, position tracking error, control torque (or control input), mismatched time-varying disturbance estimation, and matched time-varying disturbance estimation are shown in Fig. 4. During the scenario, mismatched disturbance (or called human force) is applied at the endpoint of the robot back and forth (this situation can be considered as interaction torque increase or decrease), which is generated by holding the end-effector of robotic link. One can clearly observe that the experimental results have validated the ability of mismatched disturbance rejection and the performance of trajectory tracking under the proposed continuous SMC + GPIO control strategy.

C. Test 3: GPIO-Based Continuous SMC Design Strategy Under Matched Time-Varying Disturbances

Finally, in the third experiment, we demonstrate the robust performance of the continuous SMC + GPIO controller to suppress matched time-varying disturbances. Fig. 5 describes the trajectories tracking curves of link position, position tracking error, control torque (or control input), mismatched time-varying disturbance estimation, and matched time-varying disturbance estimation. Now, we consider a time-varying disturbance, which is given by:

$$d_{21}(N \cdot m) = \begin{cases} 0, & 0 < t \leq t_1 \\ 0.038, & t_1 < t \leq t_2 \\ 0.038e^{-2\sin^2(10(t-t_2))} \cos^2(5(t-t_2)), & \text{others}. \end{cases}$$

When this matched time-varying disturbance is applied to motor side during the scenario, we observe that the proposed continuous SMC + GPIO controller presents a good performance in the accomplishment of the tracking task of the reference trajectory in spite of matched time-varying disturbance.

VI. CONCLUSION

This brief investigates the output reference trajectory tracking problem for compliantly actuated robots subjected to unknown matched/mismatched time-varying disturbances using a general robust SMC control framework. By designing a novel sliding surface, a GPIO-based continuous SMC law has been developed, which tackles matched/mismatched time-varying disturbances simultaneously. A detailed proof has also been provided to guarantee the stability of the closed-loop system. Experimental results have demonstrated that the proposed control strategy can achieve the promising tracking performance.
REFERENCES


