Online data-driven composite adaptive backstepping control with exact differentiators

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SUMMARY

This paper presents an online data-driven composite adaptive backstepping control for a class of parametric strict-feedback nonlinear systems with mismatched uncertainties, where both tracking errors and prediction errors are utilized to update parametric estimates. Hybrid exact differentiators are applied to obtain the derivatives of virtual control inputs such that the complexity problem of integrator backstepping can be avoided. Closed-loop tracking error equations are integrated in a moving-time window to generate prediction errors such that online recorded data can be utilized to improve parameter adaptation. Semiglobal asymptotic stability of the closed-loop system is rigorously established by the time-scales separation and Lyapunov synthesis. The proposed composite adaptation can not only avoid the application of identification models and linear filters resulting in a simpler control structure, but also suppress parametric uncertainties and external perturbations via the time-interval integral. Simulation results have demonstrated that the proposed approach possesses superior control performances under both noise-free and noisy-measurement environments. Copyright © 2015 John Wiley & Sons, Ltd.

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KEY WORDS: adaptive backstepping; composite adaptation; mismatched uncertainty; exact differentiator; data-driven control

1. INTRODUCTION

Adaptive integrator backstepping is an effective control strategy for a class of parametric strict-feedback nonlinear systems (SFNSs) with mismatched uncertainties [1]. Yet this control strategy suffers from the problem of ‘explosion of complexity’ caused by the repeated derivations of virtual control inputs using their analytical expressions, where the complexity of the control law grows drastically as the system order increases [2]. One solution of avoiding the complexity problem is to utilize a dynamic surface control (DSC) technique [2–4], where a first-order filter is applied to the virtual control input at each backstepping step, so that the derivation of virtual control inputs can be avoided. However, because of the use of first-order filters, filtering delay errors are introduced into DSC systems resulting in large tracking-error bounds and degraded tracking accuracy [5]. An alternative solution of avoiding the complexity problem is to utilize exact differentiators [6], where some results using sliding-mode differentiators can be found in [7–9]. Nonetheless, chattering phenomena can not be avoided for sliding-mode differentiators.

On the other hand, in the traditional adaptive control, only tracking errors are applied to update parametric estimates such that parameter convergence would be slow even input signals satisfy a persistently exciting (PE) condition. Composite adaptive control (CAC) that utilizes both tracking errors and prediction errors to update parametric estimates aims to achieve higher tracking accuracy and better parameter convergence through faster and smoother parameter adaptation [10]. In the CAC, identification models and linear filters are usually applied to generate prediction errors. The superior performance of CAC has been demonstrated in various applications, where related results
denote the minimum and maximum functions, respectively, $\text{sgn}$ the sign function, and $\| \cdot \|$ the absolute value and 2-norm, respectively. The semiglobal asymptotic stability of the closed-loop system is rigorously established by the time-scales separation and Lyapunov synthesis. The advantages of the proposed composite adaptation are as follows: (i) the application of identification models and linear filters can be avoided resulting in a simpler control structure; and (ii) parametric uncertainties and external perturbations can be suppressed via the time-interval integral.

This paper presents a novel CABC method for a class of parametric SFNSs, where the derivatives of virtual control inputs are generated by hybrid linear/nonlinear exact differentiators such that the complexity problem of integrator backstepping can be avoided, and prediction errors are generated by moving-interval integrals such that online recorded data can be utilized to improve parameter adaptation. Semiglobal asymptotic stability of the closed-loop system is rigorously established by the time-scales separation and Lyapunov synthesis. The advantages of the proposed composite adaptation are as follows: (i) the application of identification models and linear filters can be avoided resulting in a simpler control structure; and (ii) parametric uncertainties and external perturbations can be suppressed via the time-interval integral.

This rest of this paper is organized as follows. The problem under consideration is formulated in Section 2. The CABC design is given in Section 3. Illustrative examples are given in Section 4. Conclusions are provided in Section 5. Throughout this paper, $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{R}^n$ denote the spaces of real numbers, positive real numbers, and real $n$-dimensional vectors, respectively, $| \cdot |$ and $\| \cdot \|$ denote the absolute value and 2-norm, respectively, $L_2$ and $L_\infty$ denote the space of square-integrable and bounded signals, respectively, $\Omega_c := \{ \| x \| \leq c \}$ denotes the ball of radius $c$, $\min \{ \cdot \}$, and $\max \{ \cdot \}$, respectively, $\text{sgn}(\cdot)$ denotes the sign function, and $\mathcal{W}^k$ represents the space of signals for which all $(k-1)$-order derivatives have finite Lipschitz constants, where $x \in \mathbb{R}^n$, $c \in \mathbb{R}^+$, and $n$, $m$, and $k$ are positive integers.

2. PROBLEM FORMULATION

Consider a class of $n$th order generalized parametric SFNSs as follows [5]:

\[
\begin{align*}
\dot{x}_i &= f_i(x_i) + g_i(x_i)x_{i+1} + \Phi_i^T(x_i)\theta \\
\dot{x}_n &= f_n(x) + g_n(x)u + \Phi_n^T(x)\theta
\end{align*}
\]

where $u(t) \in \mathbb{R}$ and $x_1(t) \in \mathbb{R}$ are the control input and the system output, respectively, $x_i(t) := [x_1(t), x_2(t), \ldots, x_i(t)]^T \in \mathbb{R}^i$ ($x_0(t) = \mathbf{0}$) are the measurable state vectors, $f_i(x_i) : \mathbb{R}^i \mapsto \mathbb{R}$, $g_i(x_i) : \mathbb{R}^i \mapsto \mathbb{R}$ and $\Phi_i(x_i) : \mathbb{R}^i \mapsto \mathbb{R}^N$ are known functions, $\theta \in \Omega_c$, $c$ is a vector of unknown constant parameters, $N$ is the number of the unknown parameters, $c_d \in \mathbb{R}^+$ is a known constant, and $i = 1, 2, \ldots, n$. Let $x_d(t) \in \mathbb{R}$ denote a desired output. The following general assumptions presented in [32] are exploited for control synthesis.

**Assumption 1**

There is a constant $g_0 \in \mathbb{R}^+$, so that $g_i(x_i) > g_0$ with $i = 1$ to $n$, $\forall x \in \mathbb{R}^n$.

**Assumption 2**

$f_i, g_i, \Phi_i \in \mathcal{W}^{n+1-i}$ on $x \in \Omega_{c_x}$ and $x_d \in \mathcal{W}^{n+2}$ for $i = 1, 2, \ldots, n$.

Define tracking errors $e_i(t) := x_i(t) - x_i(t)$, an error vector $e(t) := [e_1(t), e_2(t), \ldots, e_n(t)]^T$, and $\Omega_{c_0}$, $\Omega_{c_x}$, $\Omega_{c_d}$ for $e(0)$, $x(0)$ and $x_d(t)$, respectively, where $i = 1, 2, \ldots, n$, $c_0 = x_d$, $c_1$ to $c_{n-1}$ are virtual control inputs, $x_d(t) := [x_d(t), \dot{x}_d(t), \ldots, x_d^{(n)}(t)]^T$, and $c_{d_0}, c_{x_0}, c_d \in \mathbb{R}^+$ are known constants. From [32], the adaptive backstepping control law is determined as follows:

\[
\begin{align*}
\alpha_1 &= \frac{1}{g_1} \left( -k_1 e_1 + \dot{x}_d - f_1 - \Phi_1^T \hat{\theta} \right) \\
\alpha_i &= \frac{1}{g_i} \left( -k_i e_i - g_{i-1} e_{i-1} + \dot{\alpha}_{i-1} - f_i - \Phi_i^T \hat{\theta} \right) \\
u &= \frac{1}{g_n} \left( -k_n e_n - g_{n-1} e_{n-1} + \dot{\alpha}_{n-1} - f_n - \Phi_n^T \hat{\theta} \right)
\end{align*}
\]

where $k_i \in \mathbb{R}^+$ ($i = 1, 2, \ldots, n$) are control gain parameters, and $\hat{\theta} \in \Omega_{c_{\theta}}$ is an estimate of $\theta$.
Let $\hat{\theta} := \theta - \hat{\theta}$ denote a vector of parameter errors and $\Phi(x) := \sum_{i=1}^{n} \Phi_i(x_i)$ denote a vector of basis functions. The control objectives of this study are to introduce proper differentiators for the estimation of $\dot{\alpha}_i$ and $\dot{\alpha}_{n-1}$ and to design a proper adaptive law of $\hat{\theta}$ such that the closed-loop system guarantees stability in the sense that $x_1$ tracks $x_d$ as fast and accurate as possible.

3. COMPOSITE ADAPTIVE CONTROL DESIGN

3.1. Differentiator-based adaptive backstepping

In the conventional adaptive backstepping control, the calculation of $\dot{\alpha}_i$ with $i = 1, 2, \cdots, n - 1$ in (2) involves the repeated derivations of the analytical expressions of $\alpha_i$ resulting in ‘explosion of complexity’, that is, the controller complexity grows drastically as $n$ increases [32]. To solve this problem, a hybrid linear/nonlinear exact differentiator is introduced as follows [35]:

$$\dot{\alpha} = \omega - \lambda_1 |\hat{\alpha} - \alpha|^{(\sigma+1)/2} \text{sgn} (\hat{\alpha} - \alpha) - \kappa_1 (\hat{\alpha} - \alpha)$$

$$\dot{\omega} = -\lambda_2 |\hat{\alpha} - \alpha|^{\sigma} \text{sgn} (\hat{\alpha} - \alpha) - \kappa_2 (\hat{\alpha} - \alpha)$$

with $\hat{\omega}(0) = \alpha(0)$ and $\hat{\alpha}(0) = \hat{\alpha}(0)$, where $\alpha \in \mathbb{R}$ is an input signal, $\hat{\alpha}, \omega \in \mathbb{R}$ are state variables, $\lambda_1, \lambda_2 \in \mathbb{R}^+$ and $\kappa_1, \kappa_2 \in \mathbb{R}^+$ are design parameters related to the linear and nonlinear parts of (3), respectively, and $\sigma \in \mathbb{R}^+$ is a design parameter related to estimation accuracy. It is worth noting that under the selection of $\kappa_1 = \kappa_2 = \sigma = 0$, the differentiator (3) is degraded to the sliding-mode differentiator in [6]. Applying $\alpha_i$ in (2) to (3) as input signals, one obtains $\dot{\alpha}_i$, the estimates of $\dot{\alpha}_i$, where $i = 1, 2, \cdots, n - 1$. Replacing $\dot{\alpha}_i$ by $\hat{\alpha}_i$ in (2), one obtains the differentiator-based adaptive backstepping control law as of the following form:

$$\alpha_1 = \frac{1}{g_1} (-k_1 e_1 + \dot{x}_d - f_1 - \Phi_1^T \hat{\theta})$$

$$\alpha_i = \frac{1}{g_i} (-k_i e_i - g_{i-1} e_{i-1} + \dot{\hat{\alpha}}_{i-1} - f_i - \Phi_i^T \hat{\theta}) \quad (i = 2, 3, \cdots, n - 1)$$

$$u = \frac{1}{g_n} (-k_n e_n - g_{n-1} e_{n-1} + \dot{\hat{\alpha}}_{n-1} - f_n - \Phi_n^T \hat{\theta})$$

The robustness against noise of the differentiator (3) has been proven in [35]. Lemma 1 of [5] is introduced as Lemma 1 of this study for the convenience of system analysis at the initialization stage $t \in [0, T_a]$ with $T_a > 0$, and Theorem 1 of [35] is introduced as Lemma 2 of this study to show the convergence property of the differentiator (3).

**Lemma 1 ([5])**

Consider the system (1) with Assumptions 1 and 2 driven by the control law (4) with (3). Given any $x(0) \in \Omega_{c_x0}$ with $c_x0 \in \mathbb{R}^+$, there exist positive constants $c_x > c_{x0}$ and $T_f > T_a$, so that the solution $x(t)$ of (1) satisfies $x(t) \in \Omega_{c_x}, \forall t \in [0, T_f)$.

**Lemma 2 ([35])**

Consider the differentiator (3) with the input signal $\alpha(t) \in \mathcal{W}^2$. For any given finite time $T_a > 0$, there exist proper design parameters $\lambda_1, \lambda_2, \kappa_1, \kappa_2 \in \mathbb{R}^+$ and $\sigma \in (0, 1)$ such that $\dot{\alpha}(t)$ is of the order $O(e^{(\sigma+1)/(2\sigma)})$, $\forall t \geq T_a$, where $e \in (0, 1)$, and $\dot{\alpha}(t) := \dot{\alpha}(t) - \alpha(t)$ is a differentiating error. Specifically, one has $\dot{\alpha}(t) = 0, \forall t \geq T_a$ if $\sigma = 0$.

For simplifying presentation, $\sigma = 0$ is considered in the following analysis. Yet similar analysis for $\sigma \in (0, 1)$ can also be obtained as exact differentiation with the order $O(e^{(\sigma+1)/(2\sigma)})$ can be

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1The condition of the control input $u$ being saturated in [5] is not presented here since the boundedness of $u$ given by (4) is naturally guaranteed by the boundedness of the state vector $x$.
guaranteed by (3) with $\sigma \in (0, 1)$. Based on Lemmas 1 and 2, the following theorem is established to demonstrate an important property of the entire system at $t \in [0, T_f)$.

**Theorem 1**
Consider the virtual control inputs $\alpha_i(t)$ in (4) with their corresponding differentiators in the form of (3) with $\sigma = 0$, where $\hat{\theta}$ is designed to be of $\mathcal{W}^2$, and $i = 1, 2, \ldots, n - 1$. If the differentiator corresponding to $\alpha_i(t)$ is implemented at $t \in [T_i, T_f)$, where $0 = T_1 < T_2 < \cdots < T_n < T_f$, then there exist proper design parameters $\lambda_1, \lambda_2, \kappa_1, \kappa_2 \in \mathbb{R}^+$ in (3) for each differentiator such that $\hat{\alpha}_i = 0$, $\forall t \in [T_i, T_f)$, where $\hat{\alpha}_i := \alpha_i - \alpha_i$ are differentiating errors.

**Proof**
The convergence analysis is carried out by the following steps.

**Step 1:** Consider $\alpha_1(t)$ in (4) at $t \in [0, T_f)$, where its corresponding differentiator is implemented at $t \in [0, T_f)$. Using Lemma 1 and Assumption 2, one gets $e_1, \hat{x}_d, f_1, g_1, \Phi_1 \in \mathcal{W}^2$ on $x(t) \in \Omega_{\epsilon \epsilon}$ with $t \in [0, T_f)$. Combining with $\hat{\theta} \in \mathcal{W}^2$, one has $\alpha_1(t) \in \mathcal{W}^2$ on $x(t) \in \Omega_{\epsilon \epsilon}$ with $t \in [0, T_f)$. Thus, Lemma 2 can be applied to state that for a given $T_2 \in (0, T_f)$, there exist suitable $\lambda_1, \lambda_2, \kappa_1, \kappa_2 \in \mathbb{R}^+$ and $\sigma = 0$ in (3) such that $\hat{\alpha}_1(t) = 0$, $\forall t \in [T_2, T_f)$. Step 2 ($2 \leq k \leq n - 1$): Consider $\alpha_k(t)$ in (4) at $t \in [T_k, T_f)$ with its corresponding differentiator implemented at $t \in [T_k, T_f)$. Because $\hat{\alpha}_{k-1}(t) = 0$, $\forall t \in [T_k, T_f)$ from Step $k - 1$, $\hat{\alpha}_{k-1}$ can be replaced by $\hat{\alpha}_{k-1}$ in the expression of $\alpha_k(t)$, $\forall t \in [T_k, T_f)$. Applying Lemma 1 and Assumption 2, one gets $e_k, \hat{x}_{d, k}, f_k, g_k, \Phi_k \in \mathcal{W}^2$ resulting in $\alpha_k(t) \in \mathcal{W}^2$ on $x(t) \in \Omega_{\epsilon \epsilon}$ with $t \in [T_k, T_f)$. Therefore, Lemma 2 can be applied to state that for a given $T_{k+1} \in (T_k, T_f)$, there exist suitable $\lambda_1, \lambda_2, \kappa_1, \kappa_2 \in \mathbb{R}^+$ and $\sigma = 0$ in (3) such that $\hat{\alpha}_k(t) = 0$, $\forall t \in [T_{k+1}, T_f)$. Step n: Consider $u(t)$ in (4) at $t \in [T_n, T_f)$. Since $\hat{\alpha}_{n-1}(t) = 0$, $\forall t \in [T_n, T_f)$ from Step $n - 1$, $\hat{\alpha}_{n-1}$ can be replaced by $\hat{\alpha}_{n-1}$ in the expression of $u(t)$, $\forall t \in [T_n, T_f)$.

\[\square\]

3.2. Composite adaptation structure

It is implied from Lemma 1 and Theorem 1 that the parameters $\lambda_1, \lambda_2, \kappa_1$ and $\kappa_2$ in (3) can be properly designed such that the fast system (3) has $\hat{\alpha}_i(t) = 0$, $\forall t \in [T_n, T_f)$ and all signals of the slow system (1) remain bounded, $\forall t \in [0, T_f)$, where $i = 1, 2, \ldots, n - 1$ [5]. Now, consider the control problem at $t \in [T_n, \infty)$. Combining (1) with (4) and using Theorem 1, one obtains the closed-loop tracking error dynamics at $t \in [T_n, T_f)$ as follows:

\[
\begin{aligned}
\dot{e}_1 &= -k_1 e_1 + g_1 e_2 + \Phi_1^T \hat{\theta} \\
\dot{e}_i &= -k_i e_i + g_i e_{i+1} - g_{i-1} e_{i-1} + \Phi_i^T \hat{\theta} \\
\dot{\epsilon}_n &= -k_n \epsilon_n - g_n \epsilon_{n-1} + \Phi_n^T \hat{\theta} \\
\end{aligned}
\]

(5)

In the traditional CAC, identification models and linear filters would be applied to generate filtered counterparts of the modeling errors $\epsilon_i(t) := \Phi_i^T(t) \hat{\theta}(t)$ with $i = 1, 2, \ldots, n$ as the prediction errors [32–34]. Instead in this section, a modified modeling error $\epsilon(t) := \epsilon(t) \hat{\theta}(t)$ is defined as the prediction error [21], where $q(t) := \sum_{i=1}^{n} q_i(t)$ and

\[
q_i(t) := \int_{t-t_{i-1}}^{t} \Phi_i(x_i(\tau)) d \tau 
\]

(6)

in which $t_{i-1} \in \mathbb{R}^+$ is an integral interval. It is worth noting that the definitions of $q_i$ to be integral types in (6) are useful for avoiding the usage of the immeasurable $\dot{e}_i$ for the calculation of the prediction error $\epsilon$ as shown in the subsequent content, where $i = 1, 2, \ldots, n$. 

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Let $\hat{\theta} := [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_N]^T$. The adaptive law of $\hat{\theta}$ is designed as follows:

$$
\dot{\hat{\theta}} = \gamma \mathcal{P} \left( \sum_{i=1}^{n} \Phi_i(x_i) e_i + k_w q e \right)
$$

(7)

where $\gamma \in \mathbb{R}^+$ is a learning rate, $k_w \in \mathbb{R}^+$ is a weight factor, and $\mathcal{P}(\bullet) = [\mathcal{P}(\bullet_1), \mathcal{P}(\bullet_2), \ldots, \mathcal{P}(\bullet_N)]^T$ is a projection operator given by Hu et al. [24]

$$
\mathcal{P}(\bullet_j) := \begin{cases} 
0 & \text{if } \hat{\theta}_j = -c_a \text{ and } \bullet_j < 0 \\
0 & \text{if } \hat{\theta}_j = c_a \text{ and } \bullet_j > 0 \\
\bullet_j & \text{otherwise}
\end{cases}
$$

To derive the computational formula of $\varepsilon$, let $c(t) := \sum_{i=1}^{n} c_i(t)$, where

$$
c_i(t) := \int_{t-\tau_d}^{t} \Phi_i^T(x_i(\tau)) \theta d\tau
$$

(8)

for $i = 1, 2, \ldots, n$. Integrating all equalities in (5) at $[t-\tau_d, t]$, one obtains

$$
\begin{cases}
    c_1(t) = e_1(t) - e_1(t - \tau_d) + \int_{t-\tau_d}^{t} \left( k_1 e_1 - g_1 e_2 + \Phi_1^T \hat{\theta} \right) d\tau \\
    c_i(t) = e_i(t) - e_i(t - \tau_d) + \int_{t-\tau_d}^{t} \left( k_i e_i - g_i e_{i+1} + g_{i-1} e_i - 1 + \Phi_i^T \hat{\theta} \right) d\tau & (i = 2, 3, \ldots, n-1) \\
    c_n(t) = e_n(t) - e_n(t - \tau_d) + \int_{t-\tau_d}^{t} \left( k_n e_n + g_{n-1} e_{n-1} + \Phi_n^T \hat{\theta} \right) d\tau
\end{cases}
$$

where the time variable $\tau$ is omitted in the expressions of the previous integral parts. Noting

$$
q^T(t) \theta = c(t),
$$

(9)

the prediction error $\varepsilon$ in (7) can be calculated by

$$
\varepsilon(t) = c(t) - q^T(t) \hat{\theta}(t).
$$

(10)

**Remark 1**

For $N$ time intervals $[t_j - \tau_d, t_j]$ with $j = 1$ to $N$, a linear equation $Q \theta = c$ can be constructed by using (9), where $Q := [q(t_1), q_2(t_1), \ldots, q(t_N)]^T$ and $c := [c(t_1), c(t_2), \ldots, c(t_N)]^T$. In the ideal case that $q(t_1)$ to $q(t_N)$ are linearly independent, $\theta$ can be accurately identified by the least-squares estimation as $\theta = (QQ^T)^{-1}Qc$ such that the parametric uncertainty in (1) can be exactly canceled out. However, in most control problems, it is difficult to find $N$ linearly independent vectors $q(t_1)$ to $q(t_N)$. In this case, the proposed adaptive law of $\hat{\theta}$ in (7) can drive both the tracking error $e$ and the prediction error $\varepsilon$ toward zero as shown in the subsequent content.

### 3.3. Stability and convergence analysis

Before presenting the main result, let’s introduce Barbalat’s lemma as follows.

**Lemma 3** ([36])

Let $e(t) : \mathbb{R}^+ \mapsto \mathbb{R}^n$ be of $L_\infty$, and $\dot{e}(t) \in L_2 \cap L_\infty$, then $\lim_{t \to \infty} e(t) = 0$.

Define $\Omega_{c_\varepsilon} := \{e(x) \in \Omega_{c_\varepsilon}, x_d \in \Omega_{c_\varepsilon} \}$ and $k_i := \min_{1 \leq l \leq n} \{k_l\}$ where $c_\varepsilon \in \mathbb{R}^+$ is a constant. The following theorem is established to show the stability result of this study.
Theorem 2
Consider the system (1) with Assumptions 1 and 2 driven by the control law (4) with (3) and (7). If \( x(0) \in \Omega_{c_{x,0}} \) and \( \hat{\theta}(0) \in \Omega_{c_{\theta}} \), then the closed-loop system achieves semiglobal asymptotic stability in the sense of \( \lim_{t \to \infty} \|e(t)\| = 0 \) and \( \lim_{t \to \infty} |\varepsilon(t)| = 0 \), where the unique solution of the closed-loop system composed of (5) and (7) is defined for all \( t \in [0, \infty) \).

Proof
Because all conditions in Lemma 1 are satisfied, one gets \( x(t) \in \Omega_{c_{x}} \) such that \( e(t) \in \Omega_{c_{e}} \), \( \forall t \in [0, T_f] \) for a given \( x(0) \in \Omega_{c_{x,0}} \). Because \( \hat{\theta}(0) \in \Omega_{c_{\theta}} \), the projection operator in (7) guarantees \( \hat{\theta}(t) \in \Omega_{c_{\theta}}, t \in [0, \infty) \) [24]. Thus, all conditions in Theorem 1 are satisfied, which implies \( \ddot{\alpha}_i = 0 \), \( \forall t \in [T_n, T_f], i = 1, 2, \cdots, n - 1 \). Choose a Lyapunov function candidate
\[
V(e, \hat{\theta}) = e^T e/2 + \hat{\theta}^T \hat{\theta} / (2\gamma)
\]
at \( t \in [T_n, \infty) \) for the closed-loop dynamics (5) with (7). Because \( e(t) \in \Omega_{c_{e}} \) and \( \hat{\theta}(t) \in \Omega_{c_{\theta}} \), \( \forall t \in [0, T_f] \), one gets \( V(T_n) \in L_{\infty} \). Differentiating \( V \) along (5) and (7) with respect to time \( t \) and using the projection operator results in [24]
\[
\dot{V}(e, \hat{\theta}) \leq -k_s \|e\|^2 / 2 - k_w e^2 \leq 0.
\]

According to the Lyapunov Stability Theorem, the previous result implies that the closed-loop system is stable in the sense of \( V(\infty) \in L_{\infty} \). Thus, one gets \( e(t), \hat{\theta}(t) \in L_{\infty}, \forall t \in [T_n, \infty) \) implying \( T_f = \infty \), and consequently, all other closed-loop signals \( x(t), q(t), \varepsilon(t) \) and \( u(t) \) are also bounded, \( \forall t \in [T_n, \infty) \). Combining with \( \varepsilon(t) \in \Omega_{c_{\varepsilon}} \) and \( \hat{\theta}(t) \in \Omega_{c_{\theta}}, \forall t \in [0, T_n] \), one concludes that all closed-loop signals are bounded, \( \forall t \in [0, \infty) \). Thus according to [37, Theorem 3.3], the unique solution \( (e(t), \hat{\theta}(t)) \) of the closed-loop dynamics (5) with (7) is defined for all \( t \in [0, \infty) \).

Integrating (12) at the time interval \([T_n, \infty)\), one obtains
\[
\begin{align*}
\left\{ \int_{T_n}^{\infty} \|e(\tau)\|^2 d\tau \leq 2(V(T_n) - V(\infty))/k_s \\
\int_{T_n}^{\infty} \varepsilon^2(\tau) d\tau \leq (V(T_n) - V(\infty))/k_w \n\end{align*}
\]

Because \( V(T_n), V(\infty) \in L_{\infty} \), one obtains \( e(t), \varepsilon(t) \in L_2 \). Because all elements at the right side of the equalities in (5) are bounded, one obtains \( \dot{e}(t) \in L_{\infty}, \forall t \geq T_n \). From (6), one gets \( \dot{q}(t) = \Phi(x(t)) - \Phi(x(t - \tau_d)) \in L_{\infty}, \forall t \geq T_n \). Noting (7), one gets \( \hat{\theta}(t) \in L_{\infty}, \forall t \geq T_n \). Thus, one has \( \dot{\varepsilon}(t) = \dot{q}(t) \hat{\theta}(t) + q(t) \dot{\theta}(t) \in L_{\infty}, \forall t \geq T_n \). Now, because \( e(t), \varepsilon(t) \in L_2 \cap L_{\infty} \) and \( \dot{e}(t), \dot{\varepsilon}(t) \in L_{\infty}, \forall t \geq T_n \), Lemma 3 [36, Lemma A.2.4] is invoked to obtain \( \lim_{t \to \infty} \|e(t)\| = 0 \) and \( \lim_{t \to \infty} |\varepsilon(t)| = 0 \). Because \( c_{x,0} \) in Lemma 1 can be arbitrarily enlarged to include all possible \( x(0) \), the stability result is semiglobally asymptotic [5].

Remark 2
Compared with the existing CABC approaches of [32–34], the major difference of the proposed CABC approach is that the modified modeling error \( \varepsilon \) is obtained by the integration of the closed-loop tracking error equations in (5) such that online recorded data can be utilized to update \( \hat{\theta} \) as shown in (7). This difference brings some advantages for the proposed approach as follows: (i) the application of both identification models and linear filters in [32–34] can be avoided resulting in a simpler control structure; and (ii) the parametric uncertainty \( \theta \) in (1) and external disturbances can be suppressed even if the convergence of \( \hat{\theta} \) is not guaranteed as shown in [21, Section 2]. The costs of the advantages of the proposed approach include the following: (i) extra memory should be used in storing online recorded \( \Phi(x) \) for the last \( \tau_d \) s time duration; and (ii) computational burden would be increased for the calculation the prediction error \( \varepsilon \). However, these costs are negligible for contemporary control units.
Remark 3
The selection of the control parameters in the proposed approach can follow the following rules:

(1) for the differentiator (3), the level of measurement noise should be considered for the increase of \( \sigma \) to improve estimation accuracy, and the balance between linear and nonlinear parts of the differentiator (3) should be considered for the selection of \( \lambda_1, \lambda_2, \kappa_1 \) and \( \kappa_2 \) [35];
(2) for the control law (4), the control gain parameters \( k_1 \) to \( k_m \) can be chosen based on the actuator limitation and control bandwidth;
(3) the increase of the integral duration \( \tau_d \) in (6) can make full use of online recorded data, but a too large \( \tau_d \) can consume large amounts of memory;
(4) the increase of the learning parameters \( \gamma \) and \( k_w \) in (7) speeds up the learning process, but \( \gamma \) and \( k_w \) that are too large may lead to serious oscillations at the control input \( u \).

4. ILLUSTRATIVE EXAMPLES

4.1. Example 1: Aircraft wing rock
Consider an aircraft wing rock model with actuator dynamics as follows [12]:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \beta_1 x_3 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_1^2 x_2 + \theta_5 x_1 x_2^2 \\
\dot{x}_3 &= -x_3/\beta_2 + (1/\beta_2)u
\end{align*}
\]

where \( x_1 \) is the aircraft roll angle (rad), \( x_2 \) is the roll rate (rad/s), \( x_3 \) is the actuator output (N), \( u \) is the actuator input (V), \( \beta_1 \) is an actuator gain, \( \beta_2 \) is an aileron time constant and \( \theta_1 \) to \( \theta_5 \) are coefficients related to angle of attack \( \alpha_A \). Noting (1), one gets \( f_1 = f_2 = 0 \), \( g_1 = 1 \), \( g_2 = \beta_1 \), \( f_3 = -x_3/\beta_2 \), \( g_3 = 1/\beta_2 \), \( \theta = [\theta_1, \theta_2, \cdots, \theta_5]^T \), \( \Phi_1(x_1) = \Phi_2(x_3) = 0 \) and \( \Phi(x) = \Phi_2(x_2) = [x_1, x_2, x_3, x_1^2 x_2, x_1 x_2^2]^T \). For simulations, set \( x(0) = [0.4, 0, 0]^T \), \( \beta_1 = 1.5 \), \( \beta_2 = 1/15 \) and \( \theta = [-0.01490, 0.04154, 0.01669, -0.06578, 0.08579]^T \) at \( \alpha_A = 21.5^\circ \) [12]. The control objective is to make \( x_1(t) \) track \( x_d(t) \) generated by applying \( x_c(t) = (\pi/12) \text{sgn}(\sin(0.2\pi t)) \) to a linear filter \( G(s) = 9/(s^2 + 6s + 9) \) with \( s \) being a complex variable.

To demonstrate superiority of the proposed approach, the adaptive backstepping control (ABC) without composite adaptation (simply set \( k_w = 0 \) in (7)) is selected as the baseline controller. The parameters selection of the baseline controller follows the rules in Remark 3, where the details are given as follows: Firstly, set \( \lambda_1 = 20 \), \( \lambda_2 = 40 \), \( \kappa_1 = \kappa_2 = 20 \) and \( \sigma = 0.2 \) for the differentiator of \( \alpha_1 \) in (3), and set \( \lambda_1 = 50 \), \( \lambda_2 = 300 \), \( \kappa_1 = \kappa_2 = 20 \) and \( \sigma = 0.2 \) for the differentiator of \( \alpha_2 \) in (3); secondly, set \( k_j = 3 \) with \( i = 1, 2, 3 \) for the control law (4); thirdly, set \( \tau_d = 10 \) s in (6); and finally, set \( \gamma = 1 \) and \( c_d = 5 \) for the adaptive law (7). Note that the setting of \( \sigma \) being no zero is used for avoiding chattering phenomena, and the selection of the previous control parameters ensures a favorable performance of the baseline controller. For the proposed CABC, set \( k_w = 10 \) and the other parameters be the same as above. Let \( \sigma_r(t) \in \mathbb{R}^+ \) satisfying \( Q(t) \geq \sigma_r(t) I \) be a minimal singular value of \( Q(t) \) at time \( t \), where \( Q(t) := \int_{t-\tau_d}^t \Phi(x(\tau)) \Phi^T(x(\tau)) d\tau \).

Simulations are carried out in MATLAB software running on Windows 7, where the solver is set to be fixed-step ode 5, the sampling time is set to be \( 1 \times 10^{-3} \) s, and other settings are kept at their default values. Simulation trajectories by the two controllers are given in Figure 1. It is observed that both controllers achieve favorable control performances with smooth control inputs [Figure 1 (a) and (c)], and the proposed CABC demonstrates better parameter convergence (reflected by a smaller norm of the parameter error \( \theta \)) at the situation of weak excitation of \( \Phi(x) \) (reflected by a small \( \sigma \)) [Figure 1 (d)]. A comparison of system errors given in Figure 2 clearly demonstrates different performances of the two controllers, where higher tracking accuracy and much smaller prediction errors are shown by the proposed CABC. Comparisons of performance indexes are given in Table I, where the proposed CABC achieves a much better performance under similar control energy [Example 1, signal-to-noise ratio (SNR) = \( \infty \)]. Simulations are also carried under SNR = 40 dB measurement noise to verify robustness against noisy measurement of the proposed approach, where simulation
results are demonstrated in Table I [Example 1, SNR = 40], and simulation trajectories are omitted here since they are very similar to those without measurement noise. It is worth noting that the advantages of the proposed approach are obtained at the cost of extra memory for the storage of online recorded $\Phi(x)$. It is also observed from the simulations that the running time by the proposed approach is a bit longer since the calculation the prediction error $\varepsilon$ increases the computational burden of the entire control algorithm.

Figure 1. Simulation trajectories by various controllers in Example 1. (a) Control performance by the baseline controller. (b) Control performance by the proposed controller. (a) Adaptation performance by the baseline controller. (b) Adaptation performance by the proposed controller.

Figure 2. System errors by various controllers in Example 1. (a) Output tracking errors. (b) Prediction errors.
4.2. Example 2: A third-order nonlinear system

To further demonstrate that the online recorded data-based composite adaptation is able to speed up parameter convergence, consider a three-order parameter SFNSs as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_1^2x_2 + x_3 + \theta_1 x_1x_2 + \theta_2 x_2 \cos(x_1) \\
\dot{x}_3 &= -x_1x_3^2 + 5(2 + \sin(x_2x_3))u
\end{align*}
\]

Figure 3. Simulation trajectories by various controllers in Example 2. (a) Control performance by the baseline controller. (b) Control performance by the proposed controller. (a) Adaptation performance by the baseline controller. (b) Adaptation performance by the proposed controller.

Figure 4. System errors by various controllers in Example 2. (a) Output tracking errors. (b) Prediction errors.
Thus, one obtains $f_1 = 0$, $g_1 = g_2 = 1$, $f_2 = x_1^2 x_2$, $f_3 = -x_1 x_3^2$, $g_3 = 5(2 + \sin(x_2 x_3))$, $\Phi_1(x_1) = \Phi_3(x_3) = 0$, $\Phi(x) = \Phi_2(x_2) = [x_1 x_2, x_2 \cos(x_1)]^T$ and $\theta = [\theta_1, \theta_2]^T$. For simulations, set $\theta = [-1.5, 2.1]^T$, $x(0) = [\pi/3, 0, 0]^T$ and $x_d(t) = (\pi/6) \sin(t)$.

The selection of the control parameters in this example is the same as that of Example 1 except $\gamma = k_w = 30$. Simulation trajectories and system error comparisons by the two controllers are given in Figures 3 and 4, respectively. It is observed that both controllers achieve favorable control performances with smooth control inputs [Figure 3 (a) and (c)], the baseline controller shows very slow convergence of $\hat{x}$ even if $\Phi(x)$ is PE [Figures 3 (b)] and the proposed CABC demonstrates much faster parameter convergence [Figure 3 (d)], higher tracking accuracy and much smaller prediction errors [Figure 4]. Comparisons of performance indexes are given in Example 2 of Table I, where it is shown that the proposed CABC also achieves a much better performance under similar control energy in this example. Again, the advantages of the proposed approach are obtained at the cost of both extra memory and increased computational burden.

5. CONCLUSIONS

This paper has developed an online data-driven CABC method for a class of parametric SFNSs with mismatched uncertainties, where semiglobal asymptotic stability of the closed-loop system is rigorously established by the time-scales separation and Lyapunov synthesis. Compared with the existing composite ADSC approaches, the proposed approach has the following unique features: (i) the complexity problem of adaptive backstepping is eliminated by hybrid exact differentiators without obvious degradation of control performances; and (ii) composite adaptation based on online recorded data is achieved without the application of both identification models and linear filters. Simulation results have shown that the proposed approach possesses superior control performances under both noise-free and noisy-measurement environments. Further work would focus on composite learning control using online recorded data [38].

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