Kinematic comparison of surgical tendon-driven manipulators and concentric tube manipulators

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ABSTRACT

Robot manipulators are increasingly used in minimally invasive surgery (MIS). They are required to have small size, wide workspace, adequate dexterity and payload ability when operating in confined surgical cavity. Snake-like flexible manipulators are well suited to these applications. However, conventional fully actuated snake-like flexible manipulators are difficult to miniaturize and even after miniaturization the payload is very limited. The alternative is to use underactuated snake-like flexible manipulators. Three prevailing designs are tendon-driven continuum manipulators (TCM), tendon-driven serpentine manipulators (TSM) and concentric tube manipulators (CTM). In this paper, the three designs are compared at the mechanism level from the kinematics point of view. The workspace and distal end dexterity are compared for TCM, TSM and CTM with one, two and three sections, respectively. Other aspects of these designs are also discussed, including sweeping motion, scaling, force sensing, stiffness control, etc. From the results, the tendon-driven designs and concentric tube design complement each other in terms of their workspace, which is influenced by the number of sections as well as the length distribution among sections. The tendon-driven designs entail better distal end dexterity while generate larger sweeping motion in positions close to the shaft.

1. Introduction

Manipulators or robot arms\textsuperscript{[1]} are increasingly used in confined space applications. Typical examples include minimally invasive surgery (MIS), such as laparoscopic surgery, Single Port Access surgery (SPA), Natural Orifice Transluminal Endoscopic Surgery (NOTES)\textsuperscript{[2–6]} and industrial endoscopic non-destructive inspection\textsuperscript{[7]}. In these applications, the manipulator needs to access and/or operate in confined spaces where obstacles are abundant. Hence, the manipulator is usually snake-like, i.e., slender and has a large number of degrees of freedom (DOFs). This allows the manipulator to be able to access the target and have adequate maneuverability around it. For surgical snake-like manipulators, the cross-sectional dimension is typically in the order of 10\textsuperscript{1} mm. These manipulators are placed on a robotic platform and their flexible bending sections are inserted into the surgical cavity via a trocar. The surgeons steer the manipulator to access to the surgical target and perform operations, such as viewing, cutting tissues, suturing, knotting, etc.\textsuperscript{[8]}. Traditional snake-like manipulators are fully articulated serial robot arms with large number of joints. Each joint is actuated by

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one or two integrated motors [9], depending on the joint type. The consequence of this design is that the robot is large in size and it is complex in both mechanical structure and control. To actuate all the joints, another approach is to separate the actuators from the robot structure, such as the tensor arm [10]. In this design, the motors are placed at the robot base, and the motion is transmitted to the manipulator via tendons. Using this design, the dimension of the manipulator can be reduced to some extent but the number of tendons needed is large, which constrains the down scaling of the robot as well. The cross-sectional dimension of these snake-like robots is typically in the order of $10^2 \text{ mm}$. Miniaturization is required for them to be applied to MIS. However, to miniaturize the size of these robots, efforts needed is tremendous. As a result, this fully articulated design is uncommon in surgical manipulators. The SPRINT robot contains two arms and each arm has 3 fully actuated joints [11]. Rigorously, it is not snake-like. The i-Snake [12] is more snake-like. It has 7 controllable joints and the diameter is 12.5 mm. However, the payload of the iSnake is very limited. This greatly restricts its applications in MIS.

To reduce the number of actuators and simplify the control, underactuation could be adopted. In an underactuated system, the number of actuators is less than the number of DOFs. As seen from nature, it is not necessary that all DOFs be actively controlled to fulfill tasks. For example, human fingers are underactuated but they can grasp objects successfully. In the past two decades, quite a number of underactuated snake-like robots or manipulators have been developed. Examples include the Elephant Trunk Robot [13], OctArm [14], Air Octor [15], the wire-driven robot [16], etc. A common feature shared by these robots is that the number of actuators employed is generally much less than the robot’s DOFs. For example, the wire-driven robot has 30 spherical joints but only six motors are used to actuate the manipulator [16]. The benefit of underactuation and moving the actuators out of the manipulator body is that both the mechanical structure and control become simpler. This alleviates the difficulty of miniaturizing the snake-like robots and pushes forward the application of snake-like robots to the field of MIS. Unfortunately, the consequence of underactuation is the reduced workspace and reduced dexterity of the manipulator, compared with full-actuated serial manipulators, which have been well studied in the literatures. Therefore, in this work, we focus on the comparison of the commonly used underactuated surgical manipulator designs.

From the literature, there are three prevailing designs for underactuated surgical flexible manipulators. The first type is the tendon-driven continuum manipulators (TCMs) [1,6,17–20], in which the backbone is a continuous structure, the actuators are placed at the base and the motion of the manipulator is controlled via tendons or cables or wires, which are similar and are all referred to as tendons in the rest of this paper. The second type is the tendon-driven serpentine manipulators (TSMs) [1,6,16,21,22]. Their structure is similar to the previous type, only the backbone structure is different. It is composed of a number of short links or vertebrae, with two successive vertebrae forming a joint. The large deformation of the backbone is generated from the small rotations of the multitude joints. To regulate the joints rotations, elastic components are required [16,23,24]. The third type is the concentric tube manipulators (CTMs), in which the backbone comprises of several nested pre-curved tubes. By rotating and translating the tubes, the shape of the backbone is controlled. In this design, the motors are also placed at the robot base [25–30]. One question arises immediately: How should we choose the design? In the literature, there are a number of comprehensive reviews on continuum robots, such as the review by G. Robinson[1], R.J. Webster[6], Burgner-Kahrs [5]. In these works, existing robot systems using these

![Fig. 1. An Example of Tendon-driven Continuum Manipulator: (a) structure and components; (b) typical tendon guiding pinholes on spacer disks.](image-url)
designs as well as their modeling, actuation, sensing, control, etc. are reviewed. However, the above question remains unanswered. As coverage, manipulability and tissue avoidance are key aspects in surgical operations, we need a comparison of these designs in terms of workspace, distal end dexterity and sweeping motion during operation. In this work, we compare these three designs at the mechanism level by using kinematic analysis. Other aspects of the manipulators using these designs are also discussed in this work. These include the scaling, payload, force sensing, reliability, etc.

The rest of the paper is organized as follows: in Section 2 the three kinds of manipulator designs and their kinematics model are summarized using a unified expression, i.e. POE. In Section 3 the workspaces are compared for the manipulators with one, two and three sections. The influence of length distribution to the workspace is also discussed. In Section 4 the distal end dexterity is compared. In Section 5, sweeping motion during operation is discussed. In Section 6, scaling, force sensing, stiffness control, etc. are discussed. Finally, Section 7 concludes the paper.

2. Review of manipulator designs

The three types of prevailing snake-like surgical flexible manipulator designs are reviewed in this section, including the structure design and the kinematics model.

2.1. Tendon-driven Continuum Manipulator (TCM)

Fig. 1 shows the illustration of the TCM design. The TCM contains a single or multiple elastic continuum backbones [18], a number of tendon spacer disks, and a number of tendons, which may also be in the form of cables or wires or slim rods [31,32]. The spacer disks are fixed to the elastic backbone and the pinholes on them are used to guide the tendons. Typical pinhole arrangements on the spacer disc are shown in Fig. 1(b). For planar sections, two tendons are used. For a 2-DOF section, usually three or four tendons are used. In a three-tendon configuration, the motion of the tendons are coupled and in a four-tendon configuration, the two directional bending can be controlled independently. Larger number of tendons, such as six or eight are used in TCMs with multiple sections. The cross-sectional area of the backbone can have various forms, including rectangle [33], circular-shape or ring-shape [34]. The backbone is segmented by the tendon knots (attachment points). Along the backbone, the range between two tendon knots is a section. Tendons are attached to the distal end of each section, guided by the spacer disks, and attached to the actuators on the other end. By pulling the tendons and/or pushing the slim rods, the backbone bends toward the contracted tendon/rod. Tendon routing is important for multi-section TCMs. Generally, the motion of tendons in the latter section is coupled with the bending configuration of previous sections. To avoid/reduce coupling, the tendons of the latter section need to pass through the neutral axis of the previous sections as in [35].

The backbone bending is affected significantly by the backbone cross-sectional area. For a rectangular cross-sectional area with high aspect ratio the backbone bends in the direction of the shorter side easily, while it is difficult to bend in the direction of the other sides. For circular cross sectional areas, the backbone can bend in all directions with equal effort. Without considering gravity and other external forces, the ideal deformed shape of the backbone after bending is a circular arc as the tendons exert a bending moment on the backbone distal end. As a result, in the kinematic modeling the piecewise constant curvature (PCC) assumption is

<table>
<thead>
<tr>
<th>Symbol</th>
<th>representation</th>
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<tbody>
<tr>
<td>$L_i$</td>
<td>Length of section i</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Bending angle of section i</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Bending direction angle of section i</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of bending sections</td>
</tr>
<tr>
<td>$J_i$</td>
<td>Jacobian matrix of section i</td>
</tr>
<tr>
<td>$J'$</td>
<td>Overall Jacobian matrix of the mechanism</td>
</tr>
<tr>
<td>$q$</td>
<td>Generalized coordinates</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Joint rotation of ith bending section (for TSM); Rotation of ith tube (for CTM)</td>
</tr>
<tr>
<td>$l_i$</td>
<td>Vertebra length of ith bending section (for TSM); Insertion length of ith tube (for CTM)</td>
</tr>
<tr>
<td>$N_i$</td>
<td>Number of vertebra of section i (for TSM)</td>
</tr>
<tr>
<td>$\sigma_j$</td>
<td>Initial curvature of jth tube at bending section i (for CTM)</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Final curvature of ith bending section (for CTM)</td>
</tr>
<tr>
<td>$K_j$</td>
<td>Stiffness matrix of jth tube (for CTM)</td>
</tr>
<tr>
<td>$g_{i-1,i}$</td>
<td>Transformation matrix between section i-1 and section i.</td>
</tr>
<tr>
<td>$\xi_i(\nu_i, \omega_i)$</td>
<td>Twist associated with bending section i, where $\nu_i$ and $\omega_i$ are twist coordinates.</td>
</tr>
<tr>
<td>(\cdot)^w</td>
<td>Wedge operator: forms a matrix in se(3) out of a given vector in $\mathbb{R}^3$.</td>
</tr>
<tr>
<td>(\cdot)^V</td>
<td>Vee operator: extract the 6-dimensional vector which parameterizes a twist.</td>
</tr>
<tr>
<td>$R$</td>
<td>Rotation matrix</td>
</tr>
<tr>
<td>$p$</td>
<td>Position vector</td>
</tr>
<tr>
<td>$i, j, k$</td>
<td>Index</td>
</tr>
<tr>
<td>Others</td>
<td>Intermediate variables</td>
</tr>
</tbody>
</table>
often adopted [6]. When the manipulator bears a large payload, the PCC assumption is invalid and the deformed shape needs be calculated from static analysis [36] or dynamic analysis [37]. In this paper, we compare the manipulators at a payload-free condition, hence the PCC assumption is adopted in the kinematic modeling.

The kinematics model of the TCM can be divided into the “actuator space - configuration space mapping” and the “configuration space - task space mapping” [6]. The “actuator space - configuration space mapping” is robot specific and the “configuration space - task space mapping” is determined by the mechanism design. Workspace and dexterity of the manipulator is mainly determined by the latter mapping, supposing the manipulator is properly actuated. Hence, in the following the first mapping is not discussed. The forward kinematics model of a TCM with $M$ sections is summarized here. The model could be represented by a number of forms and we choose to use the form of product of exponential (POE) based on twists [38]. The nomenclature is as shown in Table 1. For other forms of representation, such as using D-H parameters, Frenet-Serret frames, integral presentations, etc. please refer to the relative papers [36,39–41].

Assume the manipulator has $M$ sections. For the $i$th section the backbone length is $L_i$, the bending angle is $\Theta_i$ and the bending direction is $\Phi_i$ as shown in Fig. 2. Based on the PCC assumption and POE expression [38], the transformation matrix of the $i$th section is as per Eqs. (1)–(3).

\[
\mathbf{g}_{i-1,i}^{0}(\mathbf{\omega}_i, \mathbf{v}_i) = \exp\left(\mathbf{\xi}_i\right) \mathbf{g}_{i-1,i}^{0} = \begin{bmatrix} \mathbf{R}_{i-1,i} & \mathbf{p}_i \\ 0 & 1 \end{bmatrix}
\]

\[
\mathbf{\omega}_i = [-\sin(\Phi_i) \cos(\Phi_i) 0]^T
\]

\[
\mathbf{v}_i = L_i \left[ -\frac{1}{2} \cos(\Phi_i) - \frac{1}{2} \sin(\Phi_i) \frac{1}{\sigma_i} - \frac{1}{2} \tan(\Theta_i) \right]^T
\]

where $\mathbf{\xi}_i = \begin{bmatrix} \tilde{\mathbf{\omega}}_i & \mathbf{v}_i \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{4\times 4}$, in which $\tilde{\mathbf{\omega}}_i$ is the skew matrix associated with $\mathbf{\omega}_i$, and $\mathbf{g}_{i-1,i}^{0}$ is the initial configuration of the frame $i$ in frame $i-1$. For the section $i$, the associated twist is $\mathbf{\xi}_i(\mathbf{\omega}_i, \mathbf{\omega}_i)$. The overall transformation of the TCM can be found by using the chain rule as per Eq. (4). The distal end orientation is the rotation matrix $\mathbf{R}$ of $\mathbf{g}$ and the distal end position is represented by the last column of $\mathbf{g}$.

\[
\mathbf{g}_{0,M} = \prod_{i=1}^{M} \mathbf{g}_{i-1,i}(\mathbf{\omega}_i, \mathbf{v}_i)
\]

The Jacobian matrix of the $i$th section is $\mathbf{J}_i = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 & \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix}$, in which $\mathbf{J}_1, \mathbf{J}_2, \mathbf{J}_3$ and $\mathbf{J}_4 \in \mathbb{R}^{3\times 1}$. They are expressed as per Eqs. (5)–(8):

\[
\mathbf{J}_1 = \frac{\mathbf{\omega}_i L_i}{\sigma_i} (\cos(\Theta_i) - 1)
\]

\[
\mathbf{J}_2 = \frac{L_i}{\sigma_i^2} (\Theta_i (1 - \cos(\Theta_i)) + \tilde{\mathbf{\omega}}_i \mathbf{\omega}_i (\mathbf{\Theta}_i - \sin(\Theta_i)))
\]

\[
\mathbf{J}_3 = \mathbf{\omega}_i \sin(\Theta_i) + \tilde{\mathbf{\omega}}_i \mathbf{\omega}_i (1 - \cos(\Theta_i))
\]

\[
\mathbf{J}_4 = \mathbf{\omega}_i
\]

The overall Jacobian matrix of the TCM is:

---

**Fig. 2.** Coordinate system of the TCM.
where \( A_{d_g} \) is the adjoint transformation of \( g \). The spatial velocity of the distal end can then be expressed as:

\[
\mathbf{V}^s = \mathbf{J}^s \dot{\mathbf{q}} = \begin{bmatrix} \mathbf{v}^s \\ \omega^s \end{bmatrix} \in \mathbb{R}^{6 \times 1}
\]

where \( \mathbf{q} = [\phi, \theta]^T \) is the generalized coordinates of each section as shown in Fig. 2.

The inverse kinematics of the multi-section TCM is often solved numerically using methods such as the iterative Newton-Raphson (NR) method [42] or the Levenberg-Marquardt (LM) method [43]. The procedure is as follows: first an initial value is assigned, then apply the updating rule iteratively until the solution converges. Taking the widely used LM method as an example, the updating rule is:

\[
\mathbf{q}_{k+1} = \mathbf{q}_k + H_k^{-1} J_k^T W_k (\mathbf{q}_k - \mathbf{q}_{k-1})
\]

\[
H_k = J_k^T W_k J_k + W_N
\]

where \( w_{Ni} > 0 \) is the damping factor and \( W_k = \text{diag} \{ w_{E,i} \}, w_{E,i} > 0 \) is the weighting matrix.

2.2. Tendon-driven Serpentine Manipulator (TSM)

Fig. 3 shows the illustration of the TSM design. The TSM contains a flexible backbone and several tendons. The flexible backbone comprises of a number of sequentially linked vertebrae and an elastic beam. The articulation of the vertebrae can be revolute joints or spherical joints. When the vertebrae are connected by revolute joints with parallel axis, the backbone can only bend in a plane (1D bending). On the other hand, when all the vertebrae are connected by spherical joints, the backbone can bend in all directions (2D bending). The joints’ rotations are constrained and regulated by the elastic beam. Usually, for 1D bending, the elastic beam cross sectional area is rectangle, and for 2D bending the cross section area of the elastic beam is circularly symmetric, such as ring shape.

The tendon routing of the TSM is similar to that in the TCM. By pulling the tendon a moment is exerted on the flexible backbone and will deform the backbone to part of a regular polygon (all the joints rotate identically due to the equal constraint from the elastic beam).
beam) if all the vertebrae are identical and no external load exists. This is equivalent to the PCC assumption. Since the vertebrae are rigid, the axial deformation is usually neglected.

Suppose in section \( i (i = 1, 2, \ldots, M) \) the number of vertebrae is \( N_i \), and all the vertebrae have identical length \( l_i \). The bending angle is \( \Theta_i \) and the bending direction is \( \Phi_i \) as shown in Fig. 4. By taking the PCC assumption, the joint’s rotation in section \( i \) is \( \theta = \Theta_i/N_i \). The “configuration space - task space mapping” kinematics model of the TSM can be developed in a similar way to that of a TCM. The difference lies in the twist coordinates and Jacobian for each section. They are summarized as follows.

For section \( i \) the twist coordinates \( \xi_i, \omega_i \) are:

\[
\omega_i = [\sin(\Phi_i) \cos(\Phi_i) 0]^T
\]

\[
\nu_i = [-A_i \cos(\Phi_i) - A_i \sin(\Phi_i) B_i L_i]^T
\]

For section \( i \) the elements in the Jacobian matrix are:

\[
J_{\omega} = \begin{bmatrix}
L_i B_i (1 - \cos(\Theta_i)) + A_i \sin(\Theta_i) \\
L_i (B_i \Theta_i - A_i \sin(\Theta_i) + A_i \omega_i/L_i + B_i (\Theta_i \omega_i)) \\
A_i \omega_i (1 - \cos(\Theta_i)) \\
A_i \omega_i (1 - \cos(\Theta_i))
\end{bmatrix}
\]

where

\[
A_i = \frac{L_i (N_i - 1)}{2 N_i},
B_i = \frac{1}{2 N_i} L_i (\Theta_i - \frac{1}{2} \cos(\Theta_i)),
\frac{1}{2} \cos(\Theta_i) - \frac{1}{N_i} [1 - \cos(\Theta_i/N_i)].
\]

It should be noted that, the TCM could also be treated as a special case of TSM when the length of vertebrae \( l_i = L_i/N_i \) and \( N_i \to \infty \).

### 2.3. Concentric Tube Manipulator (CTM)

Fig. 5 gives an example of the CTM design. The CTM contains a number of nested pre-curved tubes, usually super elastic NiTi tubes [26,28]. The final shape of the CTM is the result of the overlapping of these tubes. The distal end position and orientation is controlled by translating and rotating each tube, usually actuated at the base. If the effect of torsional deformation is neglected, the CTM can be modeled as in the following.

The motion principle of CTM determines that the final shape may present multiple sections with different curvatures. Suppose for a CTM with \( N \) tubes the backbone is segmented to \( M \) sections. It should be noted that, the number of sections is greater or equal to the number of tubes, i.e. \( M \geq N \). For the section \( i \), its final curvature is denoted as \( u_i \), and \( \nu_j \) represents the initial curvature of the \( j \)th tube at section \( i \).

\[
u_j = \left( \sum_{j=1}^{N} K_j \right)^{-1} \sum_{j=1}^{N} R_j (\theta_j) K_j \nu_j
\]

where \( \theta_j \) and \( K_j = \text{diag}(E_j I_{j1}, E_j I_{j2}, E_j I_{j3}) \) are the rotation angle and the stiffness matrix of the \( j \)th tube, respectively, and \( R_j (\theta_j) \) means the rotation matrix about \( Z \) axis for \( \theta_j \). The length of section \( i \), denoted as \( s_i \), is related to the translation of each tube. The transformation from the proximal end of section \( i \) to its distal end can be expressed by a transformation matrix

\[
g_{i-1, i}(\nu, s_i) = \exp(\hat{\xi}_i(s_i))
\]

where

\[
\hat{\xi}_i = \begin{bmatrix}
\bar{u}_i \\
v_i
\end{bmatrix}
\]

and \( v_i = [0 \ 0 \ 1]^T \). This is similar to the model of TCM and TSM. The total forward kinematics can also be represented as

\[
g_{0,M} = \prod_{i=1}^{M} g_{i-1, i}(\nu, s_i).
\]

The inverse kinematics of CTM can be solved based on Jacobian matrix numerically using the NR method or LM method. The
spatial velocity of the distal end is given by
\[ \xi' = (g g^{-1})^\vee \] (22)

On the other hand, the relationship between the spatial velocity and the inputs can also be expressed similar to Eq. (10) as in Eq. (23)
\[ \xi' = J' \dot{q} \] (23)

where \( \dot{q} = [\dot{\theta}_1, \dot{t}_1, \theta_2, \ldots, \dot{\theta}_N, \dot{t}_N]^T \) is the generalized coordinates.

Using adjoint transformation [44], we have
\[ \xi' = (g g^{-1})^\vee + Ad_{R_0} (g g^{-1})^\vee + \cdots + Ad_{R_{M-1}} (g g^{-1})^\vee \] (24)
where
\[ (g g^{-1})^\vee = A_i \xi_i + \xi s_i = A_i \sum_{j=1}^N b'_i \dot{\theta}_j + \xi \sum_{j=1}^N c'_j \] (25)
and
\[ b'_i = \frac{\partial \xi_i}{\partial \dot{\theta}_j}, \quad c'_j = \frac{\partial \xi_i}{\partial \dot{\theta}_j} \] (26)

Here \( A_i \) is given by [44,45].
\[ A_i = s_i I + \frac{4 - 5 \sin(\sigma_i) - 4 \cos(\sigma_i)}{2 ||v||F} \Omega_i + \frac{4 \sigma_i - 5 \sin(\sigma_i) + \sigma_i \cos(\sigma_i)}{2 ||v||F} \Omega_i^3 + \frac{2 - 6 \sin(\sigma_i) + 2 \cos(\sigma_i)}{2 ||v||F} \Omega_i^5 \] (27)

where
\[ \Omega_i = \begin{bmatrix} \dot{u}_i \\ \dot{v}_i \\ \dot{w}_i \end{bmatrix} \] (28)
\[ \sigma_i = ||u_i||_1 \] (29)

Thus \( J' \) can be written as:
\[ J' = [J_1 \ J_2 \ \cdots \ J_i \ \cdots \ J_N] \] (30)

where
\[ J_i = \left[ \sum_{j=1}^M Ad_{R_{0j-1}} A_i b'_j, \sum_{j=1}^M Ad_{R_{0j-1}} A_i c'_j \right]. \] (31)

The inverse kinematics of CTM can be solved numerically as discussed in section II.A.

3. Workspace comparison

The reachable workspace is the set of positions which the manipulator distal end (or end effector) can gain access to [46]. The
workspace of the manipulator is also affected by the base movement, which is irrelevant to the manipulator design. Therefore, in the workspace comparison the base of the manipulators are all fixed. The workspace can be derived from the kinematics model. From the kinematic models in the previous section, for all the three types of manipulators the location of the distal end is axial symmetric about Z axis. Therefore, in the following study, the full workspace in 3D space is simulated and only the workspace in XZ plane is displayed for better visualization as illustrated in Fig. 6.

In the comparison, the workspaces of the manipulators with one section, two sections and three sections are compared respectively. A MATLAB® program is used to perform the comparison based on the previous kinematics models. The simulation parameters are listed in Table 2, Tables 3 and 4 for the three types of manipulators respectively. For TCMs, TSMs and CTMs the workspace increases with the maximum bending angle. In the systems presented in literatures, such as in [47] each section in the TCM can bend over 90°; in [16] each section in the TSM can bend up to 142.5°; in [48] the curvature of the inner tube is 0.0128 and the length is 201.6 mm (bend up to 147.9°). Therefore, in the comparison, we select the maximum bending angle of the manipulator as 180° to cover most of the cases. The length of the manipulator is selected as 240 mm, which does not influence the comparison result. In the literature, most TCMs and TSMs have evenly distributed section lengths [5,16,47]. This length distribution is adopted in this study and the influence of length distribution to the workspace is discussed later.

3.1. One section manipulator

A manipulator with one section is the simplest case. In this case, the workspace of the distal end in the XZ plane is a fixed trajectory. Fig. 7 shows the three trajectories. How the manipulator reaches positions in the workspace is illustrated in Fig. 6. In Fig. 7, trajectory of the TSM is shown by the blue dashed curve; trajectory of the TCM is shown by the green dotted curve and the red solid curve shows the trajectory of the CTM. The trajectories of TSM and TCM are very close, and the trajectory of the CTM contains two branches as the tube can advance from both sides. It is noted that at the bending limits the distal end position of TCM and CTM overlap. Also shown by the simulation is that the workspace of TCM and TSM is far away from the manipulator base (0, 0), while the workspace of the CTM contains the manipulator base. Along the X direction, the TSM can achieve the largest displacement and the CTM has the smallest horizontal displacement.

3.2. Two-section manipulator

Fig. 8 shows the workspaces of the manipulators with two sections. The workspace of the TSM and the workspace of the TCM are again very close. The shape is similar to a cap. On the other hand, the workspace of the CTM is inside the cap, with very small overlap on the boundary. By adding one section the workspaces of the three types of manipulators all have been expanded a lot. Specifically, the workspace of the TCM and TSM expand mostly inward, while the workspace of the CTM expands outward. Among all, the expansion for the CTM is larger than that of the TCM and the TSM. Similar to the previous case, the workspaces of the TSM and TCM are far away from the manipulator base and the workspace of the CTM includes the base.

3.3. Three-section manipulator

Fig. 9 shows the overlapped workspaces of the manipulators with three sections. By adding one more section, the workspaces are further increased. However, the increment is much smaller compared to that in the previous case. It is seen that the overlap is increased due to the expansion of the workspaces. The three workspaces complement each other and fill the region formed by the trajectory of the TCM and CTM in Fig. 7.

From the simulation it is noted that the workspaces of the tendon-driven manipulators are distant from the manipulator base, while the workspace of the CTM contains the base. Hence, concentric tube manipulators are more suited to tasks near the base, such as operations close to the incision. The tendon-driven manipulators are more suited for tasks distant from the base. It is also noted that the area of the workspace of the CTM is larger than that of the TCM and TSM. This is attributed to the fact that the tubes in the CTM have translational DOFs. A hybrid design would yield a workspace that cover both the base and the distal region as shown in [49,50].

Fig. 10 shows the overlapped workspaces of the TCM and CTM with one, two and three sections. Since the workspace of the TSM is very close to that of the TCM, it is omitted to avoid redundancy. This reveals that for all the three types of manipulators the workspace expansion rate is the largest when the section goes from one to two. By further increase the number of sections, the workspace can still be expanded but the rate is decreasing.

Table 2

<table>
<thead>
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<th>Simulation conditions – TCM.</th>
<th>One-section</th>
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<th>Three-section</th>
</tr>
</thead>
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<tr>
<td>M</td>
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<td>2</td>
<td>3</td>
</tr>
<tr>
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<td>120</td>
<td>80</td>
</tr>
<tr>
<td>( \Theta_i )</td>
<td>[0,( \pi )]</td>
<td>[0,( \pi/2 )]</td>
<td>[0,( \pi/3 )]</td>
</tr>
<tr>
<td>( \Phi_i )</td>
<td>[0,2( \pi )]</td>
<td>[0,( \pi )]</td>
<td>[0,2( \pi )]</td>
</tr>
</tbody>
</table>
3.4. Influence of length distribution to workspace

The influence of length distribution to the workspace of these manipulators has been rarely discussed in the literature. In this

Table 3
Simulation condition – TSM.

<table>
<thead>
<tr>
<th></th>
<th>One-section</th>
<th>Two-section</th>
<th>Three-section</th>
</tr>
</thead>
<tbody>
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<td>$M$</td>
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<td>3</td>
</tr>
<tr>
<td>$N_i$</td>
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<td>8</td>
</tr>
<tr>
<td>$L_i$</td>
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<td>120</td>
<td>80</td>
</tr>
<tr>
<td>$\theta_i$</td>
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<td>$[0, \pi/24]$</td>
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<tr>
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</tr>
</tbody>
</table>

Table 4
Simulation condition – CTM.

<table>
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<th>Two-section</th>
<th>Three-section</th>
</tr>
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<tbody>
<tr>
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<td>2</td>
<td>3</td>
</tr>
<tr>
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<td>$[0, 120*]$</td>
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<tr>
<td>$</td>
<td></td>
<td>u_i</td>
<td></td>
</tr>
<tr>
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<td>$EI/i$</td>
<td>$EI/i$</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>$[0, 2\pi]$</td>
<td>$[0, 2\pi]$</td>
<td>$[0, 2\pi]$</td>
</tr>
</tbody>
</table>

Fig. 7. Workspace comparison – manipulators with one section.

Fig. 8. Workspace comparison – two-section manipulator.
work, the effects are discussed taking the 2-section manipulators as the example.

Fig. 11 shows the workspace of a 2-section TCM or TSM with length ratio \( L_1/L_2 \) from 0.1 to 5. The total length is the same as in the previous simulation, i.e. \( L_1+L_2=240 \). From the simulation results, the length of the first section has a major influence to the overall size as well as shape of the workspace of the TCM or TSM. As shown in Fig. 11 (a) and (b), when the first section is short the manipulator has limited access to the area close to the Z axis. When the length ratio is larger than 0.5, the shape of the workspace does not change much but becoming thinner when the ratio increases. This indicates that, when designing a 2-section TCM/TSM the length ratio is better to be in the range \([0.5, 1]\).

The influence of the length distribution to the 2-section CTM is shown in Fig. 12. For the 2-section CTM, the inner tube is longer than the outer tube, i.e., \( L_1/L_2 \leq 1 \). Other simulation conditions are the same as in Table 4. From the results, the length of the outer tube also has a significant influence: the longer the outer tube, the larger the workspace. The outer tube seems to bring the inner tube close to the insertion axis. The longer the outer tube, the smaller the gap between the two branches of the workspace. When \( L_1/L_2 > 0.625 \) the workspace does not change much.

4. Distal end dexterity comparison

For a surgical manipulator, it is always desirable that the end effector, or the distal end, can reach the target position in the reachable workspace with as many configurations as possible. The more configurations, the more dexterous the manipulator is at this position. Dexterity of a manipulator can also be measured in several other ways, such as by the condition number of the Jacobian matrix [51] and by the product of the singular values [52]. However these metrics gives little intuitive information. A more intuitive way is to use the absolute kinematic flexibility \( f_a \) [46], which is defined as the number of configurations that the manipulator can reach the target. However for manipulators with more than 3 DOFs, the number of configurations may be infinite. Therefore, directly applying the absolute kinematic flexibility would also be meaningless.

The dexterous solid angle, \( D(\chi) \), is another metric for measuring the dexterity of the manipulator [53–55]. To define the dexterous solid angle, at first we assume a service sphere with unit radius is centered at the target position \( P(\chi) \); its total area is \( 4\pi \).
The manipulator can reach $P(x)$ with more than one configurations and the tangent line of the distal end intersects the service sphere at multiple points, which are called the service points. The set of all service points on the sphere form a region, named service region, as shown in Fig. 13.

Assume the area of the service region is $S(x)$, the dexterous solid angle is defined as the area ratio of the service region and the service sphere, i.e. $D(x) = S(x)/4\pi$. It can be approximated as follows: 1) divide the service sphere into $n$ by $n$ pieces; 2) assume the service points are located on $m$ pieces, then the dexterity solid angle of the manipulator at $P(x)$ is defined as per Eq. (33). When

---

**Fig. 11.** Workspace of the 2-section TCM/TSM with different length ratios.

**Fig. 12.** Workspace of the 2-section CTM with different length ratios.
it is noted that, \( D(x) = 1 \) denotes the manipulator cannot access to the position, i.e., this point is outside the reachable workspace.

\[
D(x) = \frac{S(x)}{4\pi} \approx \frac{B(x)}{n} \times 100\% \tag{32}
\]

In the following simulation, same as that in the workspace simulation, the dexterity of all the positions in 3D workspace are calculated and only the dexterity map in XZ plane is displayed due to the axial symmetry. In the simulation, the service sphere is divided into 3600 pieces. The structure of the manipulators the same as in the previous section. The simulation procedure is as follows: 1) select a target position P in the workspace and set \( m=0 \); 2) choose a direction pointing to piece \( i=1 \) on the service sphere and solve for the inverse kinematics using LM method as in section II. If valid solution exists, \( m=m+1 \); 3) repeat step 2) and iterate \( i \) from 1 to 3600; 4) the dexterous solid angle for P is then \( m/3600 \).

For manipulators with one section the distal end can only reach the target positions with one configuration when the total bending angle is limited to \( \pi \) as in the previous settings. Therefore the solid angle is always \( 1/n^2 \). For manipulators with two sections, the dexterity is also very limited. As shown by simulation, for TCM and TSM, the maximum solid angle is 0.0023 and for CTM the maximum solid angle is even smaller (0.00056). Therefore, in the following, the dexterous solid angle distribution over the workspace is presented for the TCM, TSM and CTM with three sections.

The dexterity distributions of the manipulators with three sections are shown in Fig. 12. Fig. 14 (a) shows the dexterity distribution of the three-section TCM. From the result, the dexterity is symmetrically distributed over the workspace. The maximum dexterity of the three-section TCM is 0.2103, which is achieved at the middle of the workspace. The distribution has three layers, similar to a sandwich. In the middle of the workspace the dexterity is above 0.1, and on the fringes the dexterity is between 0 and 0.1. This suggests that, when operating a TCM, it is better to have the surgical site located in the middle of the workspace.

Fig. 14(b) shows the dexterity distribution of the three-section TSM. From the result, the maximum dexterity is nearly the same (0.2101) as that of TCM. The distribution is also very similar. However, a close examination reveals that, the TCM is a little bit more dexterous than the TSM.

Fig. 14(c) shows the dexterity distribution of the three-section CTM. From the result, the distribution is also symmetric and the maximum dexterity is 0.08, which is achieved at the top left/right of the workspace. Compared with the maximum dexterity of the TCM and TSM, the CTM is 2.6 times less dexterous. At the bottom of the workspace, there is a large area that the dexterity is \( 1/n^2 \), i.e. the CTM can reach the positions with only one configuration. The reason is that the CTM accesses to this area by two sections, or the three-section CTM is degenerated to a two-section CTM. Also, the variation of the dexterity is not as smooth as in that of the TCM and TSM. This is mainly attributed to the fact that the CTM cannot control the curvature of the sections freely as the curvature depends on the tube pre-curvature and the tube overlap.

5. Sweeping motion in operation

In the previous sections, the workspace and dexterity are both related to the distal end of the manipulator. In MIS, the shape of the manipulator body is also important, especially in surgical interventions. The body of the manipulator is desired interfering with the surrounding tissues as less as possible. Especially in MIS, such as neurosurgery, the surrounding tissues are very sensitive and delicate, the body of the manipulator is desired to follow the distal end trajectory without deviation (follow-the-leader). The interfering is generated by the sweeping motion (lateral movement) of the manipulator. In this section, we compare the sweeping motion of the three manipulators in operation.

In the previous workspace and dexterity analysis, the bases of the manipulators are all fixed. In MIS, the manipulator can translate along the shaft and rotate about it. In the sweeping motion study, we add a translational DOF to each of the manipulators as follows:
where $z_b$ is the base translation along the shaft and $g_{0,M}$ is as in section II.

Since the insertion of the manipulator does not affect the sweeping motion, we only compare the sweeping motion when the tip moves laterally. In the following study, the manipulators with one section are used to follow a horizontal trajectory. Two cases are studied. In the first case, the trajectory is close to the manipulator shaft ($Z$ axis) and in the second case the trajectory is away from the manipulator shaft. The structure of the manipulators are same as in the previous sections.
Fig. 15. Sweeping motion of the manipulators during path following: the tip of the manipulator follows a given horizontal trajectory and the body configurations are displayed. In (a)–(c) the horizontal distance is within the critical distance of CTM ($L/\pi$); in (d)–(e) the horizontal distance is beyond $L/\pi$ and is within the critical distance of the TCM and the TSM ($2L/\pi$).

Fig. 15(a)–(c) shows the results of the first case. When the trajectory is close to the manipulator shaft, the area being swept by the CTM is much smaller than that of the TCM and the TSM. Also, when the length of the trajectory is less than $L/\pi$, the body of the three manipulators are all below the trajectory. Once the trajectory length is beyond the critical length ($L/\pi$), the body of the CTM crosses the trajectory while the TCM and the TSM don’t. At such circumstances, the sweeping motions of the TCM and TSM are much smaller than that of the CTM as shown by Fig. 15(d)–(f). The critical length is achieved when the bending angle of the TCM is $\pi/2$. This is the same for the TCM and the TSM. As shown in Fig. 16, once the horizontal displacement is greater than the critical distance ($2L/\pi$), the TCM and the TSM cross the trajectory as well. The maximum horizontal displacement of the TCM and the TSM is achieved when the bending angle is 2.33, which is derived from the forward kinematics model. From this example, during surgical interventions CTMs are suited to operations close to the manipulator shaft, while for TCMs and TSMs, keeping a proper distance can reduce the sweeping motion. To avoid excessive sweeping motion, the manipulators are suggested operating within their critical distance. However, when the surgery needs be performed retrospectively (anatomy below the horizontal line as in Fig. 15(f)), one should select the CTM and operate it beyond the critical distance.

Sweeping motion also exists in multi-section manipulators. Only for very limited trajectories, the “follow-the-leader” can be achieved as shown in [56]. Generally, sweeping motion cannot be eliminated for arbitrary trajectories. However, for multi-section manipulators, due to the redundancy, optimal motion planning schemes can be utilized to reduce the sweeping motion. Such as in [57] a kinodynamic planning with minimal sweeping area for TSMs is presented. Motion planning for multiple DOFs manipulators is a wide topic and is beyond the scope of this paper. Interested readers are suggested to refer to related books/papers, such as [58,59].
6. Discussion on non-kinematics related aspects

Workplace, dexterity and sweeping motion are three important aspects of a surgical manipulator. Others, such as scaling, force sensing, stiffness control, etc. also need be considered for surgical manipulators. These are discussed in this section.

6.1. Scaling

Scaling is the ability to scale up or scale down the size of the manipulator. For the TSM, the backbone deformation is achieved by joints’ rotations. Scaling up can be achieved by increasing the dimensions of the vertebrae and the joints’ rotations won’t be affected. However, scaling down the TSM is highly dependent on the manufacturing capability. The complex features on the vertebrae is a great challenge to traditional machining especially when the size is down to a few millimeters. For 3D printing, the precision is a concern. Assembly is another issue. In a TSM, the number of components is large, including tens of vertebrae, a number of tendons, and the elastic beam. The smallest TSM by far is perhaps the ureteroscope. The tip diameter of the Cobra (by Wolf Co.) is 6.0 mm, which is 2.0 mm [60]. Further scaling down is a challenge. The dimension range of the TSM is typically between several millimeters and a few decimeters. For the TCM, the number of components is much smaller. Also the structure of the spacer disks are much simpler than that of the vertebrae in a TSM. Scaling down a TCM is easier since both the fabrication and parts assembly are less difficult. However, the limit of scaling down is similar as the dimension depends on the size of the elastic backbone and tendons. To scale up the TCM, the size of the elastic backbone should also be increased. This makes the bending difficult. To ease the TCM bending, the backbone diameter cannot be too large. The contradiction is buckling when a large force is exerted on the slim backbone. The scaling range of the TCM is between several millimeters and a few centimeters. The backbone bending of the CTM is by overlapping the concentric elastic tubes. The number of components is further reduced. Also, the structure of the components in the CTM is very simple, i.e., pre-curved tubes. The fabrication and assembly is the easiest among the three. Thus, CTM is superior in scaling down. The diameter of the smallest tubes in the CTM can be down to sub-millimeters [25]. However, scaling up is difficult. The cross sectional moment of inertia is proportional to the biquadratic of the tube diameter. The torque needed to rotate the inner tubes would increase dramatically, and will intensify the snapping problem, which will be discussed later. What’s more, material fatigue is another concern. The scaling range of the CTM is typically in the range of a few hundred micrometers and a few millimeters.

It should be noted that, in the MIS, the typical range of the diameter of tools is between a few hundred micrometers and a few centimeters, depending on the place of the surgery. For surgeries in which a smaller tool is needed, such as transnasal surgery and neurosurgery, the CTM is a better option; while for surgeries in which there is a larger space, such as cardiac surgery, laparoscopy surgery and transoral surgery, the TCM and the TSM are suited to.

6.2. Intrinsic force sensing ability

Haptic feedback is important in surgeries. It relies on the force sensing ability of the robot. A traditional way of establishing haptic feedback is to place force sensors at the joints. However, for the TCM [61] and CTM [62] they have an intrinsic force sensing ability. The motions of these two types of manipulators rely on the elastic backbone bending. For the TCM, backbone deformation is controlled by the tendons and is sensitive to external forces. To maintain the deformation of the backbone the tendon forces adapt to...
external forces. For CTMs to maintain backbone deformation, the forces during tube insertion and tube rotation are adapted to external forces. By establishing the kinetostatic model, the external force exerted onto the backbone can be inferred from the backbone deformation and actuation force. For the TSM, intrinsic force sensing is also possible [63]. The deformation of the manipulator is controlled by the tendon pulling forces and affected by the external loads. From the static model, the external forces can be calculated from the manipulator bending configuration and the tendon forces. Thus, all three manipulators have the intrinsic force sensing ability but the resolutions are different, which depend on the sensitivity of the deformation of the backbone to external forces. The internal friction between the joints in the TSM and the friction between the nested tubes in the CTM affects the backbone deformation, thus impairing the force sensing resolution. For the TCM, since there are no joints and no interactions between backbones, the influence of friction (between the tendons and spacer disks) is less. Therefore, TCM has the best resolution amongst the three types of manipulators.

6.3. Stiffness control

In MIS, especially NOTES, the manipulator enters into the body through a tortuous path and performs operations around the surgical site. It is desired that the manipulator is soft when entering into the body, and lock its shape after reaching the target region [64]. In other MIS, such as laparoscopic surgery, the manipulator also needs be compliant for the sake of safety. However, during the operation, the manipulator needs adequate stiffness to fulfill the surgical tasks, such as grasping, suturing, knotting, etc. These require the manipulator can adjust its stiffness. Several static models were proposed for the TSM, TCM and CTM designs [17,63,65–67]. From the literature [61,63], the stiffness of the TSM and the TCM can be controlled by the pre-tensions in the tendons, or by increasing the tension in the tendons simultaneously. Stiffness control of the CTM is also possible. As in [68], a stiffness controller is proposed and implemented on a 3DOF CTM. However, the stiffness control in the CTM is to employ the fact that stiffness changes with backbone configuration and the amount of control is dependent on the backbone stiffness. The stiffness of the backbone is not changed. It should be noted that, this strategy is also applicable for TSMs and TCMs.

The range of stiffness which can be controlled is also important. For TCMs, one possible problem of increasing the tension in the tendons is backbone buckling (when the axial load exceeds the critical load, the backbone deforms into a buckled configuration), and the critical load for buckling is proportional to the bending stiffness. For a TCM, to ease the bending, the bending stiffness is usually small. Thus, the tensions which can be applied to the backbone is limited. This also means that the range of controllable stiffness is small. To increase the range, anisotropic material which has higher rigidity in the longitudinal direction can be applied. Also, by using slim rods as tendons, the axial load on the primary backbone can be reduced and the range can be increased. However, this improvement is limited. For TSMs as the vertebræ are rigid, the rigidity in the axial direction is much higher and it is not related to the bending stiffness. Therefore, large tensions can be applied to the backbone of the TSM. The slim rod approach is applicable for TSMs. Therefore, the range of controllable stiffness for the TSM is the largest.

6.4. Others related to safety in MIS

Besides the scaling, force sensing and stiffness control, the three manipulators all suffer from some other problems and are worth to discuss.

For TCMs and TSMs, the bending is achieved by pulling/pushing the tendons/rods. During the operation, tendon wear is unavoidable. The MIS procedure often lasts for hours, and the friction between the tendons and other components in the TCM and the TSM may break the tendons. In contrast, CTMs don’t suffer from this problem. This made the TCMs and TSMs less reliable than the CTMs. The consequence of tendon broken is loss of bending control and discontinue the surgical procedure. Another problem in TCMs and TSMs is backbone buckling when the tensions in the tendons exceed the critical force. By replacing the tendons to slim rods and bend the backbone by "push and pull" can increase the critical force. Also, buckling can be avoided by setting a safety threshold and monitoring the tendon tension.

For CTMs, one of the problem is snapping (the backbone switches immediately from one configuration to another distant configuration suddenly in an uncontrolled manner). Since the diameter of the tubes in the CTM is small and the speed during snapping is large, when snapping occurs, the tube may cut the surrounding tissues accidentally and cause permanent damage. Thus, snapping is very dangerous, especially in neurosurgery. Researchers have found some techniques to reduce this problem [69], however there are some tradeoffs, such as that the fabrication becomes much more difficult and the load capacity is impaired.

<table>
<thead>
<tr>
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<th>TSM</th>
<th>CTM</th>
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</thead>
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<td>distant from base</td>
<td>close to base</td>
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<td>good</td>
<td>limited</td>
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<tr>
<td>Others</td>
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<td>tendon wear</td>
<td>snapping</td>
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Table 5
Summary of the features of the manipulators.
7. Conclusion

In this paper, three prevailing surgical snake-like flexible manipulators are reviewed and compared mainly from the kinematics point of view. The results are summarized in Table 5. The TCM and TSM have similar workspace, dexterity and sweeping motion. The workspace of the tendon-driven manipulators and concentric tube manipulators complement each other well, as the former is distant from the base and the other is close to the base. A hybrid design would yield the widest workspace. The workspace is influenced by both the number of sections and length distribution. For tendon-driven designs the workspace expands inward while for CTM the workspace expands outward when the number of section increases. Measured by the dexterous solid angle, the tendon-driven manipulators are more dexterous than the CTMs, and the TCM is slightly better than the TSM. From a scaling point of view, the CTM is better suited for miniaturization while the TSM is best in scaling up. All the three types of manipulators have certain force sensing ability, with the CTM and the TCM more sensitive than the TSM. From the stiffness control point of view, the TSM is superior amongst the three designs. Related to safety, the tendon-driven manipulators suffer from tendon wear and the CTM suffers from snapping problem.

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