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Generalized Dynamic Predictive Control for Non-Parametric Uncertain Systems with Application to Series Elastic Actuators

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Abstract—One weakness of model predictive control (MPC) method is that the predicted states/outputs are constructed by an exact nominal model. Its accuracy varies if uncertainties exist, which will ultimately deteriorate the closed-loop control performances. To this end, we propose a generalized dynamic predictive control (GDPC) method for a class of lower-triangular systems subjected to non-parametric uncertainties. Instead of relying on the inherent robustness property of the standard predictive controller or on/off-line parameter identification, a dual-layer adaptive law is designed to estimate the lumped effect of system uncertainties. As another main contribution, under a less ambitious but more practical control objective, namely semi-global stability, various nonlinearity growth constraints utilized in the existing related methods could be essentially relaxed. Numerical simulation and illustrative experimental tests of a series elastic actuator (SEA) system are provided to demonstrate both simplicity and effectiveness of the proposed method.

Index Terms—Continuous-time predictive control, variable receding horizon, non-parametric uncertainty, adaptive control, semi-global stability, motion control systems.

I. INTRODUCTION

MODEL predictive control (MPC) method, originated in the field of process control [1]–[5], has been widely acknowledged as one of the most promising control approaches and is receiving a great deal of attention in engineering, such as motion control [6]–[10] and power electronics [11]–[14]. As its name suggested, however, the nominal model plays a fundamental role in the phase of prediction, i.e., its accuracy significantly affects the closed-loop control performances [15].

Unfortunately, since system uncertainties inevitably exist in real-life plants, it is always difficult to obtain an exact nominal model with high accuracy [16]. One democratic practical approach is the off-line identification or modeling before implementation [4], [5], [17], which is only suitable for time-invariant system. Besides, although the standard MPC method shows certain robustness against perturbations [18], [19], its control performances are somehow conservative since it is designed based on the worst case. Another more effective approach is adaptive MPC methods. In [20]–[23], recursive least-square estimator is embedded into the conventional MPC method for linear systems with unknown parameters. In [24], a gradient-based adaptive law is combined into MPC method for unconstrained discrete-time linear systems with parametric uncertainties. Based upon a finite-time parameter estimation technology [25], an adaptive MPC method is proposed for constrained nonlinear systems subjected to parametric uncertainties in [26]. Generally speaking, most of the existing adaptive MPC results for uncertain systems are closely related to on-line identification technologies, i.e., an additional update mechanism with at least $n$ ($n \in \mathbb{N}^+$) dimensions is designed to on-line identify parameters and real-timely update the controller for the system with $n$ unknown parameters [20]–[26]. In the case where the plant is with large number of unknown parameters, those approaches will be much complex in the form and inevitably cause huge computational resource. Furthermore, those above mentioned adaptive MPC methods are only appropriate for systems with parametric uncertainties.

However, non-parametric uncertainties are more generally reported in industrial applications. For example, as covered in [27]–[30], part of uncertainties in permanent-magnet synchronous motor drives, e.g., the effects of mechanical and electrical parameter uncertainties, can be expressed as the form of $x^T \theta$; whilst another part, e.g., dead-time effects in inverter modules and flux harmonics and cogging torques in motor bodies, can be expressed as $\sin(x^T \theta + \varphi)$. This kind of non-parametric uncertainty is exceptionally difficult to be estimated, especially when its structure is even unavailable. In addition, the problem of computational burden can not be ignored when one attempts to apply MPC methods to real-life plants with fast dynamics (e.g., power electronics and motor drivers) rather than process control communities [12], [31].

In this paper, motivated by the above mentioned challenging issues, we are aiming to develop a generalized dynamic predictive control (GDPC) approach for a family of single-input...
single-output nonlinear systems with non-parametric uncertainties. First, considering the nominal chain of integrators, a conventional nonlinear generalized predictive controller (GPC) [32] is designed. Second, inspired by the dynamic high-gain technologies [33]–[35], a dual-layer update law is designed in order to determine a suitable predictive period for the system subjected to non-parametric uncertainties. Compared with the previous related adaptive MPC results, the main contributions of this paper are summarized by the following two aspects:

i) The structural restriction of uncertainties in the considered system can be relaxed from strict parametric form to a non-parametric one.

ii) Only an additional dual-layer update law is required, which is simpler in the form than identification based adaptive MPC methods.

The remainder of this paper is organised as follows. Several useful lemmas and the problem formulation are provided in Section II. The main design framework with rigorous semi-global stabilization analysis is covered in Section III. Position control results of a series elastic actuator (SEA) system with various comparative experimental studies to the conventional GPC method [32] are provided in Section IV. Finally, a conclusion is drawn in Section V. For brief of expressions, the following notations are given in priori.

- The symbols \( \mathbb{R} \), \( \mathbb{R}^+ \), \( \mathbb{N} \) and \( \mathbb{N}^+ \) denote the real number set, the positive number set, the natural number set and the positive integer set, respectively.
- The symbols \( C^i \) (\( i \in \mathbb{N} \)) denote the sets of all differentiable functions whose first \( i \)-th time derivatives are continuous.
- \( \forall A \in \mathbb{R}^{\alpha \times \beta} \), \( \lambda_{\text{max}}(A) \) and \( \lambda_{\text{min}}(A) \) denote the maximum and the minimum eigenvalues of \( A \).  
- \( \forall i, j \in \mathbb{N} \) and \( i \leq j \), \( \mathbb{H}_{i,j} = \{i, i+1, \ldots, j\} \).
- \( \forall x_i \in \mathbb{R}^n \), \( \forall i \in \mathbb{N}_{1:n_1} \), \( \bar{\xi}_i = [x_1, x_2, \ldots, x_i]^T \) and \( x \leq \bar{x} \).
- \( \forall x \in \mathbb{R}^n \), \( ||x|| \leq \sqrt{x^T x} \) and \( ||x||_{\infty} \leq \text{max}(|x_1|, |x_2|, \ldots, |x_n|) \).

II. Preliminaries and Problem Formulation

A. Useful Lemmas

The following technical lemmas are crucial in the development of the main result of the paper.

**Lemma 1 (36):** For any real-valued continuous function \( f(x, y) \) where \( x \in \mathbb{R}^m \), \( y \in \mathbb{R}^n \), there exist two smooth scalar functions \( a(x) \geq 1 \) and \( b(y) \geq 1 \) such that \( |f(x, y)| \leq a(x)b(y) \).

**Lemma 2 (Barbalat’s Lemma):** If \( \lim_{t \rightarrow \infty} f(t) < \infty \) exists and if \( \dot{f} \) is uniformly continuous (or \( \dot{f} \) is bounded), then \( \lim_{t \rightarrow \infty} \dot{f}(t) = 0 \).

B. Problem Formulation

In this paper, a family of single-input single-output nonlinear systems with non-parametric uncertainties is considered and depicted by

\[
\begin{align*}
\dot{x}_i &= x_{i+1} + \phi_i(\theta, \tilde{x}_i), \quad i \in \mathbb{N}_{1:n_1} \\
\dot{\tilde{x}}_i &= u + \phi_i(\theta, x) \\
y &= x_1
\end{align*}
\]

(1)

where \( x_i \) (\( i \in \mathbb{N}_{1:n_1} \)), \( u \) and \( y \) are the system states, the control input and the controlled output, respectively; \( \theta \in \mathbb{R}^p \) is an unknown parameter vector and is assumed to be within a known boundary, i.e., \( \theta \in [\bar{a}, \bar{a}^\theta] \) where \( a \) and \( \bar{a} \) (\( a < \bar{a} \)) are known constants; \( \phi_i : \mathbb{R}^p \times \mathbb{R}^{n_i} \rightarrow \mathbb{R} \) (\( i \in \mathbb{N}_{1:n} \)) are \( \mathbb{C}^\Gamma \) uncertain nonlinear functions.

Owing to the presence of non-parametric uncertainties, it is worth pointing out here that the exact regulation objective for system (1) is almost impossible to be achieved via continuous control design. In this paper, for rigorous theoretical presentation, the proposed method is illustrated by first solving an exact asymptotical stabilization problem for system (1) under a vanishing condition (i.e., \( \phi_i(\theta, 0) = 0 \)), whilst a practical tracking result of a given time-varying reference can also be obtained in parallel by several modifications introduced in [37], which will be demonstrated in the experimental results in Section IV.

With this aim, this paper is to solve an optimal problem of the system (1), i.e., design a controller such that the closed-loop system is asymptotically stable and what’s more, the output \( y(t) \) of the uncertain system (1) optimally converges to the origin in terms of the following receding-horizon performance index

\[
J(t) = \frac{1}{2} \int_t^{T} y(t + \tau)^2 \, d\tau
\]

(2)

where \( T > 0 \) is the predictive period.

The above mentioned problem will be solved by a variable predictive period \( T = T(0)/L, \ T(0) > 0 \) where \( L \) is its changing rate determined by the following dual-layer update law

\[
\begin{align*}
\hat{c} &= \alpha(L, x), \quad \alpha : \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^+ \\
L &= \beta(L, \hat{c}), \quad L(0) = 1, \quad \beta : \mathbb{R}^+ \times \mathbb{R} \rightarrow \mathbb{R}^+.
\end{align*}
\]

(3)

By solving the minimum problem of (2) with a variable receding-horizon (3), the proposed controller is deduced as

\[
u^* = \psi(T, x), \quad \psi : \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}.
\]

(4)

The closed-loop system (1)-(3)-(4) can be rendered to be semiglobally stable, which will be discussed in Section III.

**Remark 1:** Under a less ambitious but more practical control objective, namely semi-global stability [38], [39], a dynamic high-gain control technology is incorporated into the conventional GPC method. Original restrictive nonlinearity growth condition in the dynamic high-gain control technology is not necessary any more (see, e.g., [36, Assumption 3.2] and [40, Assumption 2.1]).

III. Main Results

In this section, a step-by-step procedure is presented to design the proposed GDPC law for system (1) with rigorous semi-global stability analysis.

A. Output Prediction Based on the Nominal Model

By ignoring the non-parametric uncertainties \( \phi_i(\theta, \tilde{x}_i) \) (\( i \in \mathbb{N}_{1:n} \)), the nominal system of (1) is obtained as follows

\[
\dot{x}_i = x_{i+1}, \quad i \in \mathbb{N}_{1:n_1}, \quad x_n = u.
\]

(5)
Along system (5), the future output  \( \hat{y}(t+\tau) \) within the predictive period (i.e.,  \( 0 \leq \tau \leq T \)) is predicted by using the following Taylor series expansion

\[
\hat{y}(t+\tau) \Big|_{(s)} \approx x_1 + x_2 + \cdots + \frac{\tau^{n-1}}{(n-1)!} x_n + \frac{\tau^n}{n!} u(0) + \cdots + \frac{\tau^{n+r}}{(n+r)!} u(r) 
\]

where

\[
\mathcal{T} = \begin{bmatrix} 1, \tau, \tau^2, \ldots, \tau^{n-1} \end{bmatrix}, \quad \hat{\mathcal{T}} = \begin{bmatrix} \tau^n, \tau^{n+1}, \ldots, \tau^{n+r} \end{bmatrix} \quad \text{and} \quad \mathcal{U} = \begin{bmatrix} u, u^T, \ldots, u^T \end{bmatrix}^T
\]

and \( r \in \mathbb{N} \) is named as the control order [32].

### B. Receding-Horizon Optimization

With the assistance of (6), the performance index (2) can be predicted as

\[
\hat{J}(t) = \frac{1}{2} \int_0^T \hat{y}(t+\tau)^2 d\tau \\
= \frac{1}{2} x^T \mathcal{T}_1 x + x^T \mathcal{T}_2 u + \frac{1}{2} u^T \mathcal{T}_3 u
\]

where

\[
\mathcal{T}_1 = \int_0^T \hat{\mathcal{T}}^T \mathcal{T} \hat{\mathcal{T}} d\tau \in \mathbb{R}^{nxn}, \quad \mathcal{T}_2 = \int_0^T \hat{\mathcal{T}}^T \mathcal{T} \mathcal{T}_2 d\tau \in \mathbb{R}^{nx(r+1)} \\
\mathcal{T}_3 = \int_0^T \hat{\mathcal{T}}^T \mathcal{T} \mathcal{T}_3 d\tau \in \mathbb{R}^{r+1}(r+1)
\]

and partial derivative of \( \hat{J} \) with respect to \( U \) yields \( \partial \hat{J} / \partial U = \mathcal{T}_2^T \mathcal{T} x + \mathcal{T}_3 U \). Note that the matrix \( \mathcal{T}_3 \) is positive definite. Letting \( \partial \hat{J} / \partial U = 0 \) and \( \partial^2 \hat{J} / \partial U^2 > 0 \), the optimized control sequence is obtained as \( \mathcal{U}^* = -\mathcal{T}_3^{-1} \mathcal{T}_2^T x \). Taking the first row of the control sequence, the implementable GDPC law is given by

\[
u^* = -I \mathcal{T}_3^{-1} \mathcal{T}_2^T x
\]

where \( I \in \mathbb{R}^{nxn} \) denotes the identity matrix. The dual-layer update law in the form of (11) is

\[
\dot{\hat{c}} = \frac{\rho_2}{d} \| z \|^2 \\
L = \rho_1 L \max(0, \hat{c} - \rho_4 L), \quad L(0) = 1
\]

where \( \rho_1, \rho_2, \rho_3 \) and \( \rho_4 \) are tunable parameters satisfying

\[
\rho_1 > \max \left \{ 0, -\frac{\lambda_{\min}(DP + PD)}{2n\lambda_{\max}(P)} \right \}, \quad \rho_2 > 0 \\
\rho_3 \geq \frac{-2\lambda_{\min}(P)\rho_1 + \lambda_{\min}(DP + PD)}{2n\lambda_{\max}(P)} > \rho_4 > 0
\]

The derivation of (11) and the definitions of \( \hat{c} \) and \( d \) are all left in the next subsection.

**Remark 2:** The dual-layer update law in the form of (11) is only provided for rigorous semi-global stability proof. For practical applications, a new variable \( \gamma = \hat{c} \) is suggested for simplicity, as shown in Fig. 1.

**C. Dynamic Predictive Period Design**

Different from fixing the predictive period in the conventional GPC method [32], a dynamic approach is employed in this paper. Let \( z_i \in \mathbb{R}^{L_i} \) (\( i \in \mathbb{N}_{1:n} \)) where \( L_i \) is an auxiliary design parameter which will be determined later. In the re-scaled coordinate, system (1)-(9) can be then compactly expressed as

\[
\dot{z} = LAz - (\rho_1 I + D) \frac{L}{L^*} z + \Phi(\theta, x, L)
\]

where

\[
z = [z_1, z_2, \ldots, z_n]^T \\
D = \text{diag}[0, 1, \ldots, n-1] \\
\Phi = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_n \\ \frac{L_0}{L_{i+1}} & \frac{L_0}{L_{i+2}} & \cdots & \frac{L_0}{L_{i+n-1}} \end{bmatrix} \\
A = \begin{bmatrix} \frac{L_0}{L_1} & \cdots & \frac{L_0}{L_{n-1}} & \frac{L_0}{L_n} \end{bmatrix}
\]

\( I \in \mathbb{R}^{nxn} \) denotes the identity matrix. According to [32, Theorem 2], the stability of the nominal system (5)-(9) only depends on the control order \( r \), i.e., by choosing \( r \) appropriately, there exists a positive definite and symmetric matrix \( P \in \mathbb{R}^{nxn} \) satisfying \( A^TP + PA = -I \).

In what follows, a dual-layer update law for the predictive period is firstly presented in the form as

\[
\begin{bmatrix} \hat{c} \\ L \end{bmatrix} = \begin{bmatrix} \rho_2 \|z\|^2 \\ \rho_1 L \max(0, \hat{c} - \rho_4 L), \quad L(0) = 1 \end{bmatrix}
\]

where \( \rho_1, \rho_2, \rho_3 \) and \( \rho_4 \) are tunable parameters satisfying

\[
\rho_1 > \max \left \{ 0, -\frac{\lambda_{\min}(DP + PD)}{2n\lambda_{\max}(P)} \right \}, \quad \rho_2 > 0 \\
\rho_3 \geq \frac{-2\lambda_{\min}(P)\rho_1 + \lambda_{\min}(DP + PD)}{2n\lambda_{\max}(P)} > \rho_4 > 0
\]

\( \rho_4 \) is a tunable parameter. The derivation of (11) and the definitions of \( \hat{c} \) and \( d \) are all left in the next subsection.

**Remark 2:** The dual-layer update law in the form of (11) is only provided for rigorous semi-global stability proof. For practical applications, a new variable \( \gamma = \hat{c} \) is suggested for simplicity, as shown in Fig. 1.

---

**Fig. 1.** The implementation block diagram of the proposed GDPC method.

**Remark 3:** Unlike the performance index in the MPC method, how to penalize the control energy is not explicitly included in the conventional nonlinear GPC method (see, e.g., [8], [11], [31], [32], [41]–[43]). As suggested in [32], however, this energy can still be saved by choosing low control order \( r \) or large initial predictive period \( T(0) \), which will be demonstrated in Section IV [see Fig. 7].
Remark 4: In the conventional dynamic high-gain control methods, the choice of initial control gains is relatively conservative, i.e., not only Hurwitz, but also several matrix inequalities are required to be satisfied (see, e.g., [34, Theorem A1] and [35, Lemma 1]). Although the existence of control gains is guaranteed by constructive proof, it still brings obstacle to practices since those gains are chosen based on stability without considering any other control performances. One improvement of the proposed GDPDC method is that only Hurwitz condition is necessary for the initial gains, making the optimization possible to be achieved.

D. Semi-Global Stability Analysis

Theorem 1: Consider the closed-loop system (1)-(9)-(11) satisfying \( (x(0)^T, \dot{c}(0))^T \in \Gamma \equiv [-p, p]^{n+1} \) where \( p \) is a positive constant that could be arbitrarily large. The following statements hold.

i) All the signals in the closed-loop system are uniformly bounded, i.e., \( \lim_{t \to \infty} x = 0 \).

Proof: By Lemma 1, for each \( C^0 \) function \( \dot{a}_j / \partial x_j, \) \( i \in \mathbb{N}_{1,n}, \) \( j \in \mathbb{N}_j, \) there exist smooth scalar functions \( a_{ij}(\theta) \geq 1 \) and \( b_{ij}(\xi) \geq 1 \) such that \( \dot{a}_j / \partial x_j \leq a_{ij}(\theta) b_{ij}(\xi). \) Let \( \varepsilon = \max(\varepsilon_1(\theta), a_{ij}(\theta), \ldots, a_{ij}(\theta)). \) Noting that \( c \) is an unknown constant dependent on \( \theta, \) one gets that \( c \in [\hat{c}, \bar{c}] \) where \( \hat{c} \) and \( \bar{c} \) are known constants.

Construct the Lyapunov function as \( V(z, \tilde{e}) \equiv W(z) + \varepsilon^2/2(\varepsilon_1) \) where \( W(z) \equiv z^T P \varepsilon, \varepsilon_1 > 0 \) is a design parameter, \( \tilde{e} \equiv c - \hat{c} \) and \( \hat{c} \) is the estimate of \( c. \) For any initial states satisfying \( (x(0)^T, \dot{c}(0))^T \in \Gamma \equiv \tilde{c}(0) \in [c - \rho, c + \rho], \)

\[
\begin{align*}
\Omega_\varepsilon & \equiv \{z^T \in \mathbb{R}^{n+1} \mid V(z, \tilde{e}) \leq M\} \\
\Omega_\varepsilon & \equiv \{[\hat{c}, \bar{c}] \in [c - \rho, c + \rho]\}
\end{align*}
\]

where \( M \equiv \max \{z \in [-\varepsilon_1, \varepsilon_1]^T \mid \varepsilon_1 \in \mathbb{R}_{\geq 0}\} \) and \( N \equiv \max \{z \in [-\varepsilon_1, \varepsilon_1]^T \mid \varepsilon_1 \in \mathbb{R}_{\geq 0}\} \).

For organized proof, the following four steps are given.

(a) \( \forall (x^T, \dot{c}) \in \Omega_\varepsilon, \exists \varepsilon \in \{0, 1\}, \)

\[
\text{s.t.,} \quad V(z, \tilde{e}) \leq -L(1 - \varepsilon_3)^2/2. \]

Taking the derivative of \( W(z) \) along system \( (10) \) gives that

\[
W(z) \bigg|_{(10)} = -L\|z\|^2 + 2\dot{\Phi}^T P \varepsilon - L \varepsilon_1^2 (2\rho \varepsilon + DP + PD). \]

The last two terms on the right-hand side of \( (13) \) will be firstly estimated below.

(a) With the assistance of Mean-Value Theorem, the following relation holds for \( x_i \neq 0, i \in \mathbb{N}_{1,n} \)

\[
\begin{align*}
\left| \frac{\phi_i(\theta, x_i)}{L^{n-1+i}} \right| & = \frac{1}{L^{n-1+i}} \left| \phi_i(\theta, x_i) - \phi_i(\theta, 0) \right| \\
& = \frac{1}{L^{n-1+i}} \left| x_i \cdot \frac{\partial \phi_i}{\partial x_i} \bigg| x_i = x_i = \xi \right| \\
& \leq \frac{c}{L^{n-1+i}} \left| x_i \right| \cdot \left| x_i \right| \cdot \max(b_{i,1}(\xi), b_{i,2}(\xi), \ldots, b_{i,n}(\xi))
\end{align*}
\]

where \( \xi_i \in (0, x_i) \cup (x_i, 0). \) For all \( x \in \Omega_N, x_i \neq 0, \) we have

\[
\left| \frac{\phi_i(\theta, x_i)}{L^{n-1+i}} \right| \leq \frac{cd}{L^{n-1+i}} \left| x_i \right| + \ldots + \left| x_i \right|
\]

\[
= \frac{1}{L^{n-1+i}} \left| x_i \right| + \ldots + \left| x_i \right| \leq \frac{1}{L^{n-1+i}} \left| x_i \right| + \ldots + \left| x_i \right| \leq \frac{1}{L^{n-1+i}} \left| x_i \right| + \ldots + \left| x_i \right| \leq \frac{1}{L^{n-1+i}} \left| x_i \right| + \ldots + \left| x_i \right|
\]

\[
\leq \frac{1}{L^{n-1+i}} \left| x_i \right| + \ldots + \left| x_i \right| \leq \frac{1}{L^{n-1+i}} \left| x_i \right| + \ldots + \left| x_i \right| \leq \frac{1}{L^{n-1+i}} \left| x_i \right| + \ldots + \left| x_i \right| \leq \frac{1}{L^{n-1+i}} \left| x_i \right| + \ldots + \left| x_i \right|
\]

where \( d \equiv \max \{b_{i,1}(\xi), b_{i,2}(\xi), \ldots, b_{i,n}(\xi)\} \geq 1 \) is a known constant. Noting that \( (14) \) still holds for any \( x_i = 0, \) one then gets that

\[
\left \| 2\dot{\Phi}^T P \varepsilon \right \|_{\Sigma \leq 2ncd\lambda_{\text{max}}(P)} ||z||^2. \]

(b) By (a), (b) and \( y = \hat{c} \varepsilon, \) one can conclude that both \( z \) and \( \gamma \) are uniformly bounded. In addition, since \( y \) is monotonically increasing, the limitation of \( \gamma \) exists. Letting \( \bar{y} \equiv \lim_{y \to y} \gamma < \infty, \) we have \( y \leq \bar{y} \) for \( t \in [0, \infty), \)

Next, by choosing a sufficiently large parameter \( \rho_1 \) satisfying \( (12), \) we show that \( 2\rho_1 + DP + PD \) can be rendered positive definite. Keeping in mind that \( P \) is positive definite, one gets that

\[
z^T(2\rho_1 + DP + PD)z \geq e_2 ||z||^2 \geq 0
\]

where \( e_2 \in (0, 2\lambda_{\text{min}}(\rho_1 + \lambda_{\text{min}}(DP + PD))). \)

Submitting both \( (15) \) and \( (16) \) into \( (13) \) yields

\[
W(z) \bigg|_{\Sigma \leq 2ncd\lambda_{\text{max}}(P)} \leq 2ncd\lambda_{\text{max}}(P) ||z||^2 - e_2 ||\varepsilon||^2.
\]

By (17), one arrives at (18) on the top of the next page where \( e_2 \in (0, 1) \) is a design parameter. Let \( \varepsilon_2 = \rho_2 / (2\lambda_2^2 \lambda_{\text{max}}(P)), \)

\[
eq \frac{2\lambda_2^2 \lambda_{\text{max}}(P)}{\rho_3} \varepsilon_2 \leq \frac{2\lambda_2^2 \lambda_{\text{max}}(P)}{\rho_3} \varepsilon_2 \leq 2\lambda_2^2 \lambda_{\text{max}}(P)\rho_4. \]

Taking (11) into (18) yields that (a) holds.

(c) Uniform Boundness of Variables—Statement i).

By (a), (b) and \( y = \hat{c} \varepsilon, \) one can conclude that both \( z \) and \( \gamma \) are uniformly bounded. In addition, since \( y \) is monotonically increasing, the limitation of \( \gamma \) exists. Letting \( \bar{y} \equiv \lim_{y \to y} \gamma < \infty, \) we have \( y \leq \bar{y} \) for \( t \in [0, \infty), \)

Next, a contradiction argument is used again to prove the uniform boundness of \( L. \)

Keeping in mind that \( L \) is increasing, if \( L \) is not bounded, we know that \( \lim_{t \to \infty} L = \infty, \) which implies that \( \forall \varepsilon \in \mathbb{R}, \exists \varepsilon_3 > 0, \)

\[s.t., \quad \lim_{t \to \infty} L(t) > \varepsilon.\]

Without loss of generality, we choose \( \gamma = \bar{y} / \rho_4. \)

There are two possible cases needed to be discussed below.

Case ci — \( \gamma \leq \rho_4. \)

\[
\rho_4 L(t) = \rho_4 L(t) = \rho_4 \gamma \leq \gamma \leq \gamma \text{ holds for } t \geq 0, \text{ which results in that } \forall t \geq 0, L \equiv L(0) = 1 \text{ [see Fig. 3(a)].}
\]
\[ V(z, \tilde{c})|_{z \in \Omega_N} = W - \frac{\tilde{c} \tilde{c}}{\varepsilon_1} \leq -L(1 - \varepsilon_3)||c||^2 - \frac{\varepsilon_2}{L} \left( L - \frac{2n \lambda_{\max}(P)}{\varepsilon_2} \right) \left( \hat{c}d - \frac{\varepsilon_3}{2n \lambda_{\max}(P)} L \right) \|c\|^2 - \frac{\varepsilon}{\varepsilon_1} \left( \hat{c} + 2nd \lambda_{\max}(P) \|c\|^2 \right) \]

where \( x_i (i \in \mathbb{N}_{1:3}) \) and \( u \) are the system states and the control input, respectively; parameter vector \([\theta_1, \theta_2, \theta_3] = [1.5, 2, 3]\) is unknown to the designer.

Since the system uncertainties in (20) are non-parametric, most identification approaches including [4], [5], [17], [25] can not be applied. Besides, even in the case where all the parameters are available as priori knowledge, several nonlinear methods associated with feedback linearization [38] also can not be applied in this example since the relative degree of \( x_1 \) is not well-defined.

In the simulation, the control order \( r \) is chosen as 0 for simplicity. By calculating (8), one gets the optimized control gains as \([k_1', k_2', k_3'] = [10, 8, 4, 3]\). Other parameters and initial conditions are listed as follows: \([\rho_1, \rho_2, \rho_3, \rho_4] = [10, 0.6, 25, 0.04], [T(0), L(0), \gamma(0)] = [1.5, 1, 0]\) and \( x(0) = [1, 0, -1]^T\).

As shown in Fig. 4, all the signals are bounded and system states converge to the origin asymptotically. By Fig. 4(b), one gets that the control input is relatively small, i.e., \( u \in (-12, 10) \).

In Fig. 4(c), the predictive period \( T \) starts to decrease at 32.6[ms] and finally converges to 0.5303.
To show the superiorities of the proposed method, system (21) is controlled by and compared with the conventional recursive least-square (RLS) estimator based adaptive GPC method [22]. In Example 2, the system initial conditions and the parameters of the proposed controller are chosen as the same ones in Example 1. The initial conditions of the RLS estimator are chosen as \([\hat{\theta}_1(0), \hat{\theta}_2(0), \hat{\theta}_3(0)] = [0, 0, 0]\) and \(P(0) = I \in \mathbb{R}^{3 \times 3}\).

As shown in Fig. 5(b), the predictive period \(T\) of the proposed GDPC method finally converges to 0.55. In Fig. 5(a), the curves of all the system states and the control inputs under the GDPC method and the conventional adaptive GPC method with different predictive periods (1.50 and 0.55) are provided. Compared with the proposed GDPC method, the convergence rate of the conventional adaptive GPC method is poor when its predictive period is set as 1.50. Although all the system states converge faster when the predictive period is set as 0.55, its control input is too large, i.e., \(u \in (-60, 30)\). The validity of the RLS estimator is shown in Fig. 5(c).

IV. APPLICATION TO ROBUST POSITION CONTROL OF SEA

In this section, experimental studies of a series elastic actuator (SEA) system are implemented to validate the feasibility and effectiveness of the proposed GDPC method.

Due to their superiorities over conventional stiff actuators in force control, SEAs have been extensively applied to plenty of advanced compliant robots, e.g., “Valkyrie” in national aeronautics and space administration (NASA) and “Lopes” in University of Twente [44]. Compared with the force control, the position control problem of a SEA is more complicated than that of the conventional stiff actuator, which is therefore regarded as a benchmark in this section.

A. Dynamic Model Description and Controller Design

By using Newton’s law of motion, the following nominal model of SEA is obtained [44]

\[
\begin{align*}
\begin{cases}
m_m \ddot{q}_m + b_m q_m &= F_m - k(q_m - q) \\
m_l \ddot{q}_l + b_l q_l &= k(q_m - q)
\end{cases}
\end{align*}
\]

(22)

where \(q_m\) and \(q_l\) are the angles of the motor and link, respectively; \(F_m\) is the motor torque; \(m_m (2.2 \times 10^{-6} [\text{kg-m}^2])\) and \(m_l (4 \times 10^{-6} [\text{kg-m}^2])\) are the inertias of the motor and link, respectively; \(k (0.14 [\text{N-m/rad}])\) is the stiffness of torsional spring; \(b_m\) and \(b_l\) are the viscous friction coefficients of the motor and link, respectively. It is noteworthy that the viscous friction coefficients of the SEA are unknown, which results in that most model-based control methods are difficult to be realized with desired performances unless identifying parameters firstly.

Letting \(x \triangleq [x_1, x_2, x_3, x_4]^\top = [q_l - q_{ref}, \dot{q}_l, k(q_m - q_{ref})/m_l, k_{d_m}/m_m]^\top\) and \(u \triangleq kF_m/(m_m m_l)\) where \(q_{ref} (q_{ref} = 0)\) is the reference signal of the link angle, system (22) can be rewritten in the form of (1). Then, it is straightforward to utilize the proposed controller.

B. Experimental Results

In this subsection, position control results of a SEA [see Fig. 6] are provided. The tested SEA has two series elastic elements: a linear spring with a low stiffness and a torsional spring with a high stiffness. In this paper, we verify the proposed controller using only the torsional spring. Fig. 6 is a cross section of the used SEA. The motor [EC-4-pole brushless DC motor with Elmo Gold Whistle Servo Drive] operating at 200W is coupled to a ball screw through a torsional spring. Two incremental encoders [Renishaw RM22IC with resolutions of 2048 and 1024 pulses per revolution] are used to measure the angular displacements/velocities of the motor shaft and lead screw, respectively. The control algorithm is real-time implemented at 1kHz on the dSPACE DS1007 processor board.

In what follows, the conventional GPC method [32] is...
used to design the position controller for comparisons. The controller of GPC method has the same form of the proposed controller but with the time-invariant predictive period. By choosing the control order \( r \) as 0, the optimized gains are obtained as \([k_1, k_2, k_3, k_4] = [43.2, 36, 15.4, 4.5]\). Other parameters are listed as follows: \([\rho_1, \rho_2, \rho_3, \rho_4] = [10, 0.4, 10, 0.5]\) and \([T(0), L(0), \gamma(0)] = [9.6 \times 10^{-2}, 1, 0]\). To fairly evaluate the control performances, the following three cases are designed.

1) Case I—Set-Point Tracking Performance Tests: Figs. 7 and 8 present the set-point tracking curves of angle and torque under GDPC and GPC methods when the reference angles are 0.5[rad] and 1[rad], respectively. In Fig. 7, the effects of different \( T(0) = (9.60[ms], 9.90[ms], 10.20[ms]) \) in the GDPC method are tested, i.e., at the precondition of the same other parameters, smaller \( T(0) \) results in faster convergence rate, higher tracking precision, but larger control energy; and vice versa. As shown in Fig. 7, the predictive period \( T \) starting from 9.60[ms] decreases to 3.10[ms] in the steady-state when the reference is 0.5[rad]. The performances of the GPC method with different predictive periods, i.e., the initial one and the eventual one of the GDPC result, are tested. It is observed from Fig. 8 that the convergence rate and the tracking precision are poor when 9.60[ms] is set as the predictive period of the GPC method whilst the SEA seriously oscillates due to the control limitation protection when 3.10[ms] is set. The essential reason is that in the case where the initial predictive period is set too small (which results in the relatively large initial gains), since the tracking error is also large at the starting phase, the initial control input could be relatively large, making the output have faster convergence rate but possibly oscillate.

Detailed quantitative data for performance comparisons of Case I are given in Table I. It is worth noting that although the settling time of the link angle under the GPC method is smaller than that under the GDPC method in some cases, the offset errors under the GPC method in the steady-states are much larger in these cases, i.e., 9.67[deg.] vs. 0.17[deg.] in the case of 0.5[rad] reference and 2.46[deg.] vs. 1.05[deg.] in the case of 1[rad] reference.

2) Case II—Robustness Tests Against Artificial Uncertainties: To specially and quantitatively verify the robustness performance against uncertainties, artificial ones are added into the designed controller, i.e., \( F_{m}^{**} = F_{m}^* + \phi(t) \) where \( F_{m}^{**} \) is the real torque applied to the SEA, \( F_{m}^* \) is the control input based on the proposed method and \( \phi(t) \) is the artificial uncertainty. Figs. 9 to 12 present the response curves of angle, torque and uncertainty when \( \phi_1(t) = 27.6 \theta_1 x_1^5[\text{mN-m}], \theta_1 = 0.1 \) or 1, \( \phi_2(t) = 27.6 \theta_1 \sin(\theta_2 x_1)[\text{mN-m}], \theta_1 = 0.1, \theta_2 = 1 \) or 10, respectively. Here, 27.6[mN-m/A] is the torque constant of the used brushless DC motor. After injecting artificial uncertainties, one can conclude from the comparisons with Figs. 7 and 8 that it is relatively difficult to achieve desired performances. However, compared with the GPC results with fixed predictive periods, the advantages of the GDPC method are still evident. By Figs. 9 and 11, one can get that when the system uncertainty become more serious, the eventual predictive period in the steady-state will be smaller to provide enough robustness.

More detailed quantitative performance indexes of Case II are provided in Table II. Note that the link angle of the SEA under the GDPC method maintains almost the same performances when the system uncertain become more serious.

3) Case III—Practical Trajectory Tracking Performance Tests: Figs. 13 to 16 present the practical trajectory tracking curves of angle and torque under GDPC and GPC methods when the reference angles are 0.5 sin(5t + \( \phi \))[rad] and sin(10t + \( \phi \))[rad], respectively. It is observed from the results of the GDPC method that when the predictive period begins to decrease, the tracking precision will become higher. Compared
TABLE I
PERFORMANCE INDEXES OF CASE I.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>GDPC</td>
<td>0.60–1.10</td>
<td>0.23</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>GPC</td>
<td>0.60</td>
<td>0.84</td>
<td>10.34</td>
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<tr>
<td></td>
<td></td>
<td>10.20–2.30</td>
<td>1.23</td>
<td>10.72</td>
</tr>
<tr>
<td>1</td>
<td>GDPC</td>
<td>0.60</td>
<td>0.11</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>GPC</td>
<td>0.60</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Fig. 9. Case II—Set–Point tracking performances of the proposed GDPC method with artificial uncertainty $\phi_{4,1} = 27.661 \times 0^\circ$ [mN·m].

with the GPC results with the initial predictive period, the superiority of the proposed method on the tracking performances is obvious. Meanwhile, although the tracking precisions are similar in the steady state when the predictive period is chosen as the eventual one, the control energy of GPC method in the starting phase is much larger than that of GDPC method. The integral of squared error (ISE) index is also introduced in Table III for quantitative comparisons.

V. CONCLUSION

This paper has presented a systematic optimal approach for a class of nonlinear systems with non-parametric uncertainties. A performance index with a variable predictive period has been proposed, which leads to the robustness property against both parametric and non-parametric uncertainties. The semi-global stability of the closed-loop system has been rigorously developed. A SEA system has been conducted to illustrate the feasibility and efficacy of the proposed GDPC method. It has been shown that the proposed approach exhibits remarkable performances as compared with the conventional GPC method.

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Fig. 12. Case II—Set-Point tracking performances of the conventional GPC method with artificial uncertainty $\phi_{42} = 27.6\theta_1 \sin(2\theta_1 t)$ [mN•m].

Fig. 13. Case III—Practical trajectory tracking performances of the proposed GDPC method when the reference is $q_{ref} = 0.5 \sin(5t + \phi)$[rad].

Fig. 14. Case III—Practical trajectory tracking performances of the conventional GPC method under different predictive periods when the reference is $q_{ref} = 0.5 \sin(5t + \phi)$[rad]. (a): $T = 9.60$[ms]. (b): $T = 6.75$[ms].

<table>
<thead>
<tr>
<th>Uncertainty [mN•m]</th>
<th>Method</th>
<th>Predictive Period [ms]</th>
<th>Setting Time [s]</th>
<th>Disturb Error [deg.]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GPC</td>
<td>2.86</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>GPC</td>
<td>2.86</td>
<td>0.06</td>
<td>0.95</td>
</tr>
<tr>
<td>2.76sin(1.2)</td>
<td>GPC</td>
<td>9.60</td>
<td>2.69</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>GPC</td>
<td>9.60</td>
<td>2.69</td>
<td>-</td>
</tr>
<tr>
<td>2.76sin(10.1)</td>
<td>GPC</td>
<td>9.60</td>
<td>3.61</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>GPC</td>
<td>9.60</td>
<td>3.61</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Table II: Performance Indexes of Case II.
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Fig. 15. Case III—Practical trajectory tracking performances of the proposed GDPC method when the reference is $q_{ref} = \sin(10t + \varphi) [\text{rad}]$.

Fig. 16. Case III—Practical trajectory tracking performances of the conventional GPC method under different predictive periods when the reference is $q_{ref} = \sin(10t + \varphi) [\text{rad}]$. (a): $T = 9.60 [\text{ms}]$. (b): $T = 4.10 [\text{ms}]$.

Table III

<table>
<thead>
<tr>
<th>Reference [rad]</th>
<th>Method</th>
<th>Predictive Period [ms]</th>
<th>ISK c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.5\sin(10t + \varphi)$</td>
<td>GDPC</td>
<td>9.60</td>
<td>6.75</td>
</tr>
<tr>
<td>$\sin(10t + \varphi)$</td>
<td>GPC</td>
<td>4.10</td>
<td>2.53</td>
</tr>
<tr>
<td>$\sin(10t + \varphi)$</td>
<td>GDPC</td>
<td>148.51</td>
<td>24.98</td>
</tr>
</tbody>
</table>

c: $\text{ISE} = \int_{t_1}^{t_2} (q_{ref} - q_{act})^2 dt$ where $[t_1, t_2]$ is a period of the reference signal in the steady state.


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